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ABSTRACT

This document opens with a brief discussion of the views of Jean Piaget and his contributions to psychology. The material moves into a presentation of several research studies that have attempted to look at large and representative samples of adolescent pupils, and presents data for comparison in a cross-cultural context. The main sections cover: (1) Background to the Problem; (2) The Mathematical Structure of Formal Thought; (3) The Research Frontier; and (4) The Educational Implications. These sections are followed by five appendices, titled: (1) The Piagetian Journey; (2) The Puzzling Ways of Science; (3) Characteristics of Abstract as Compared to Concrete Thought; (4) Age Ranges for Certain Piagetian Concepts; and (5) The Modification of Concrete Structures into Formal Structures. It is hoped that this material may be useful to research workers and helpful to both teachers and teacher educators. It is felt that the appendices provide sufficient material for beginners to understand the basic Piagetian concepts in an elementary manner. (MP)

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The Mathematical Structures Underlying Adolescent Thought

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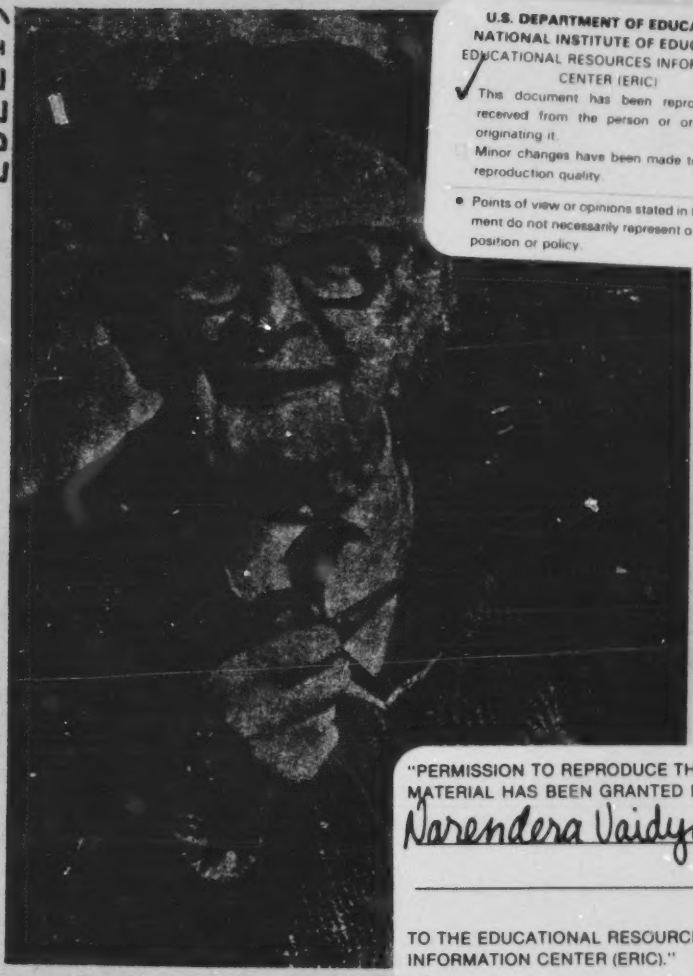

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THE MATHEMATICAL STRUCTURES UNDERLYING ADOLESCENT THOUGHT

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Stars Both

Galileo gazed at all the stars
In his 17th century sky.
Many he dismissed as a farce.
And some he looked in the eye

He lent his support, ever bleary,
To the Copernican theory.
Many see him full of hope.
And keen and eager as a child.
The first to use a telescope.
He was thought to be somewhat wild.

He had to suffer for his views.
And that, for science, was bad news.

Now he is risen from the dead.
As people speak of him anew.
Did he see Neptune from his bed ?
Or from a more proper venue ?
But nothing is certain although
Neptune must have seen Galileo.

V.D.T.
Quoted from Times of India
Sept. 15, 1980

"I have forgotten the word I intended to say, and my thought, unembodied returns to the realm of shadows."

(O. Mandlestam)

Tney do not understand the words
I speak, and my thought, unembodied
returns to the realm of shadows.

I do not know the words to say,
and my thought, unembodied
returns to the realm of shadows.

Quoted From

Mother Tongue to English

by

Daphne M. Brown

The Splitting of Jean Piaget !

The late Professor Jean Piaget became a legendary figure in his own life time. The ever-lasting nature of his work is now being acknowledged in several countries of the world for he began to split off due to the elephantine nature of his work. He is Piaget Number One when one considers his slipshod work on cognition. He is Piaget Number Two if one considers his truly scientific work on perception. He is Piaget Number Three when one considers the educational implications arising out of his work. He is Piaget Number Four when he begins to revise his ideas very fast in the light of after thought, criticism, contradictory data and self critical reflection. He is Piaget Number Five when he lifts a particular problem from its lowest plane to its highest epistemological plane whereby a sort of parallelism is established the way the idea was historically developed on the one hand and the way it arose scientifically in the child's mind right from its moment of inception. He is Piaget Number Six in the eyes of those who imagine that the basic Piagetian tenets cannot be proved wrong. He is, in the end, Piaget the Miscellaneous when he flourishes on misunderstandings raked up by those who do not understand him, or accuse him of not solving those problems which he did not propose to do.

N. Vaidya
in
The Piagetian Journey
(1896-1980)

FOREWORD

Prof. Narendera Vaidya has presented in this book several research studies conducted by him as well as his students on the Mathematical Structures of Adolescent Thought in the Piagetian Context. He has also referred to research studies conducted in this area elsewhere. In his own research, he has used the factor analytic technique and identified a number of factors involved in Cognitive thought during adolescence.

This book has attempted at some length the main Piagetian problem which seems crucial in studying the ever lasting effects of education: locating large and representative samples of adolescent pupils who have been exposed to differing amounts of schooling in the cross culture context. It is hoped that the special problem yet to be posed may require consideration at depth by the authors later on.

The publication may be useful to research workers in this area and may also be helpful to teachers and teacher educators in adopting proper strategies for concept development. For the beginners, the appendices provide sufficient material for understanding the basic Piagetian concepts in an elementary manner.

New Delhi
2nd February, 1981

R. C. Das
Prof. of Education
Head, Department of Teacher Education,
N. C. E. R. T.

ABOUT THE BOOK

The late Prof. Jean Piaget, is now being ranked the greatest psychologist of this century. Having been trained as a zoologist, he tried to move his thought in every possible direction. In the context of his own vast epistemological abstract frame of reference, not only complex to comprehend but also equally difficult to befriend, anyone who came close to his work left his own main occupation to study, reflect and interpret the most incomprehensible man of this century, for, using unscientific methods, kicking any potential worker in the face, contrary to the spirit of science, he was ultimately seen of value who did talk something relevant to the education of children, outweighing, in the process, not only the present educational thinkers but also all those who had preceded him. Handling his own confusion, and making it less opaque in the light of criticism and rereflecting in the context of varying his variables maximally, he founded more or less single-handed a new branch of knowledge called Experimental Epistemology, and the world famous Geneva school as well. With his vigorous brain-power and long hours of sitting at his desk for about fifteen hours a day for over sixty years, he began to solve an age-old problem as enunciated by Plato : 'How and when can man be sure that his knowledge is true knowledge, when time and again he finds that what he took to be knowledge has proved to be error.' With little regard for psychometry (perception excluded), he came up with ideas which ran orthogonal to the current thinking and fashion of the day. Frequently, he used to say that he was little interested in psychology, education and children. About the slow and grudging acceptance of his work, which hardly clicked contextually because of intense criticism of his work, he stated very clearly the very source of confusion among his readers which was invariably missed for several years. To quote him :

I am a naturalist and biologist by training interested in epistemological problems without ever having undertaken formal study (nor passed any examination) in psychology, my most central concern has always been to determine the contributions of person's activities and the limiting aspects of the object in the process of acquiring knowledge. Fundamentally, it was the wish to resolve this problem using the experimental approach that brought me into the field of developmental psychology. But it follows logically since this point of view is not often held by psychologists in general and even less by child psychologists that those who read my work find themselves confused.

Imagination, ingenuity and skill coupled up with inexhaustible commitment opened up several perspectives for the age-old problem already hinted at. He did not hesitate asking big questions considered definitely out of bounds by the twentieth century psychology and provided experimental answers to the philosophical questions contrary to the past philosophical tradition, beginning with Socrates. He and his co-workers contributed abundantly with commitment and passion to the whole field of psychology: perception, reasoning, intelligence, dreams, moral development, space, time, speed, play, thinking from early childhood to late adolescence, grasp of unconscious, success and understanding and other highly varied problem areas relating to mental development in physics, mathematics, biology and logic. Failure in understanding him theoretically, but not his experiments which stemmed from it, interestingly enough, made him world-famous. Jean Piaget, it is little known, had active association with initial intelligence testing in Paris in 1920 under the guidance of Prof. Binet. He first intuitively and, later on, intentionally got interested in analysis of errors which children made at

different age levels while engaged on fellowship for standardizing the well known reasoning tests of Prof. Burt. He perhaps then imagined that the nature of inquiry relating to I. Q , which was yet in the offing, will not go further than the construct or concept in comparison to the scheme of thought he was to derive from biology for understanding the origin of knowledge within the context of life-sciences. To quote R. R. Skemp and Jenson :

> It is therefore not with the starting points of the psychometric models of intelligence, but the ways in which these have been developed, that I differ; and with the fact that unfruitful lines of thinking have been persisted in for so long. In some aspects one might be pardoned for suggesting that they had gone backward; for the early pioneers did not make statements like the following :
>
> To object to this procedure by arguing that the I Q. cannot be regarded as interchangeable with intelligence, or that intelligence cannot really be measured, or that I. Q. is not the same as intelligence is to get bogged down in a semantic morass. *It is equivalent to arguing that a column of mercury in a glass tube cannot be regarded as synonymous with temperature....* (Jenson, 1970, italics mine).

It appears that in the pathless territory, the initial paths, when pursued at depth, either multiply or divide or disappear or merge depending upon the nature of the starting point taken in the context of the journey of a thousand miles in the phraseology of Confucius. Considering the short history of psychology, less than hundred years old, it has to solve its own definitional problems in the more or less similar manner as was done by the naturalists, priests turned scientists, inventors, technologists, scientists and philosophers, until the Tribunal of Experimental

Science was born in the seventeenth century which became the turning point of a series of revolutions starting with mechanics, chemistry, biology and explosion of knowledge in our century. It is not, however, suggested that the spin off of advantages should not be exploited from other branches of knowledge but that it is possible to explore any field of inquiry with tools having reliabilities and validities approaching to zero as was done by S. Freud in the study of psychoanalysis not long ago and with scant regard for the same in measurement by Piaget in the area of cognition, recently. It is said that the essence of science is to understand, comprehend, control and predict. Its other role, to contradict what is already established is little known. Unlike it, the essence of technology is the designing of new and novel products. Tradition being peculiar, these products develop as well with as without the knowledge of science. To illustrate this, the concept of ship grew from the manually driven boat with sails which in turn arose out of a simple observation made by a curious observer who saw a piece of wood floating on water in Scotland. Archimedes came too late on the scene. In the history of thermometry, physicists, not only obtained different temperatures for the same body by using different temperature measuring devices but also by using thermometers open at the top which measured simultaneously both temperature and barometric pressure, strictly speaking in the name of, temperature alone until the error was detected by Pascal in France and immediately rectified in later measurements.

The Psychology, in consistent with the spirit of science even having diverse living material, must need more or less the same criteria within its context, for developing reliable and valid knowledge which really works rather than exists. Otherwise, rephrasing Mircia Eliade, it may not be said of us that we are all very very knowledgeable people living

like the primitives in the continuous present where nothing really new and novel is happening over the centuries because, of our 'refusal to accept ourselves as historical beings' an observation contrary to the work of Charles Darwin who showed us a way of describing and explaining the continual emergence of novelties in the living world one more novel than the other in their historical context from generation to generation.

As I look back over the past two decades or so, there is a historical genesis to this problem being groped in darkness. In 1958-59, chronologically speaking, in the earlier tradition of Vocabulary and Concept counting studies in U.S.A. I began to count scientific concepts in the Primary Science Text Books like an elementary school child at the Central Institute of Education, Delhi. This, more or less a physical effort in the absence of any definition of concept, led nowhere except accounts-keeping because it utterly lacked insight into concept development among young children. Nothing better being available, the same exercise in the wider frame of reference was extended to the Outcomes of Science Education for the first eight years of schooling at the Teachers College, Vidya Bhavan, Udaipur in 1959-61. In 1961-64, at the Institute of Education, London, this old exercise was dropped and another one was formulated: A Study of Science Concepts in Relation to Comprehension Level During Adolescence. As the encounters with the problematic territory increased, the definition of the term : 'Comprehension Level' became a casualty. The problem was then shifted to the area of problem solving, production thought, precisely speaking in the pharaseology of Gestalt psychology, a School of Thought already too much rooted in philosophy. Interestingly enough, the whole problem, as it progressed, began to gravitate towards the Piagetian

psychology because it offered naturally highly mobile intellectual structures encountered during problem solving when approached individually. As if the utter confusion was not sufficient, the final problem was investigated at the formal stage of mental development with little concern for the psychometric techniques. The factorial interpretation of data led nowhere. Even today, the situation is equally hopeless, for the INRC structure is yet to be intensively investigated and interpreted factorially.

What was really the source of trouble ? The terms like fact, concept, law and principle were dropped. Instead, the term : Scheme of Thought within the context of physical sciences was finally settled at, which psychologically speaking made the investigation a bit more meaningful despite its poor understanding. To quote R. R. Skemp

(i) A schema is a structure of connected concepts. The idea of a congnitive map is a useful introduction, a simple particular example of a schema at one level of abstraction only, having concepts with little or no interiority, and representing actuality as it has been experienced. A schema in its general form contains many levels of abstraction, concepts with interiority and represents possible states (conceivable states) as well as actual states.

Factors important for effectiveness include

(ii) Relevance of content to the task in hand (rather obviously, but not always met).

(iii) The extent of its domain.

(iv) The accuracy with which it represents actuality.

(v) The completeness with which it represents actuality within this domain.

(vi) The quality of organization which makes it possible to use concepts of lower or higher order as required, and to interchange concepts and schemas. (The vari—focal part of the model, linked with the idea of interiority).

(vii) By a high-order schema, we mean one containing high-order concepts (Clearly it will not contain *only* high-order concepts.) This determines its generality, and thus the extent of its domain.

(viii) The strength of the connections.

(ix) The quality of the connections, whether associatives or conceptual.

(x) The content of ready-to-hand plans, which remain intergrated with the parent schema.

(xi) Penetration—the degree to which it can function in highnoise conditions.

(xii) Assimilatory power—the degree to which it can assimilate new experience.

(xiii) Assimilatory power relative to other schemas. Many breakthroughs of human thought have taken the form of the mutual assimilation of two schemas: e. g. radio astronomy, electomagnetism, algebraic geometry.

To conclude the discussion, there are currently several models in competition : psychometric model, Piagetian model, matching of schemata with environmental conditions model, neurophysiological model and the cybernatical one. Out of these, the Piagetian model is the most extensive as well as the intensive one. In his scheme of work, he has abundantly shown that a child is not a little adult. In other words, there are distinct qualitative differences between the two thoughts. Metamorphically speaking, children and adults are in the category of tadpole to frog undergoing metamorphosis rather than kitten and cat category on the factors of physical features and behaviour. Keeping in mind the balance of theoretical choice out of the several choices available, the present document attempts to achieve the following purposes :

1. It indicates the existence of several mathematical structures underlying adolescent thought which appeared in studies undertaken in USA, UK and India. The current picture in the context of other models is yet to appear.

2. Learnability or Educability considerably depends upon intelligence. So, human learning is not only a function of intellectual structures alone. Whereas the concept of motivation confuses our intellectual understanding of the problem, concentration, commitment, passion and patience etc. play their part significantly in achievement. The concept of I. Q. as such did not clarify the basic problems because the character of research over it led us away from the fields of experimental psychology, diverse learning theories and ultimately to failure to inform intelligently the day to day activities of the practicing teachers.

3. The present document tentatively suggests factorial structures of Pigetian tasks along with personality variables, as well as immediate test reactions to the problems on presentation, strictly speaking, in the problem solving context.

4. It stresses the necessity of developing the Mathematical model of mental development right from its moment of origin to its final development. Not only the departures and deteriorations in old age but also Hump Effect as encountered in Problem Solving concepts may also be studied, for, a schema may act both forward, backward or reversibly, for, it inheres a concept at its centrality in the phraseology of R. R. Skemp.

5. It also stresses that for structuring learning experiences as suggested by Prof. Karplus or Models of Teaching as based on Piaget's work. It is equally necessary to investigate firmly through research the psychological growth of concepts like Operation, Equilibration, INRC structures, Ego centrism, Aspect Character (Van de Ger) and Functional Fixedness (Karl Duncker) to mention a few. Moreover if fundamental concepts in the nature of Schemes of Thought, as defined psychologically within the individual conceptual schemes and defined within a given discipline or across the disciplines, are integrated together, a fruitful body of knowledge may result for stimulation of intellec-

tual thought under diverse conditions. Examples are those of Energy, Potential Entropy and Inverse Square Law etc.

6. It provides at one place sufficient material gathered by the authors empirically as well as culled and collated from different sources. The developing picture also suggests the type of future studies to be undertaken in this area for model building on Piagetian studies undertaken cross-culturally.

7. It also provides for the beginners in the appendix sufficient information about the vastness of Piaget's work: The Piagetian Journey, Puzzling Ways of Science, Age Ranges for the Various Concepts which develop naturally regardless of school influence and the Transformation of Concrete Thought into Formal Thought.

8. In this area, several models are competing. The Piagetian model, according to our view happens to be the deepest when it comes to the education of children based upon their natural characteristics. Why ? Because the human mind looks for extreme variability. It has to be readied systematically, imaginatively, persistently and reflectively for learning. It is given to the human mind to identify, to name, to order, to compare, to contrast, to construct, to demonstrate, to state a rule, to apply a rule and even to form and use a concept, if not an algebaric symbol. How the universal child goes about in the business of building the incomplete house of knowledge is illumined by the work of the late Prof. Jean Piaget. So, innovative ideas with or without research have always a long time a-borning. It is therefore necessary to keep firmly in mind, structurally speaking, the whole ball of wax before innovative practices are introduced according to every new situation in all possible directions by destroying the ball of wax in the end. It is equally true of schooling conditions in this country where there is a lot of stress on reproductive knowledge as well as evaluation hooked to the downgrad-

ing of content sequencelessly in the name of modernization of knowledge at the cost of productive knowledge and skill.

9. Lastly, it presents within the Piagetian context an incomplete body of knowledge, fusing, in the process, the psychometric and developmental aspects of intelligence.

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I am equally obliged to Dr. S. N. Dutta, our Principal, who in his inimitable style provided us every possible facility on asking.

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Jan. 25, 1981 *N. Vaidya*

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CHAPTER 1

Background to the Problem

Introduction

Since Man has always been a ceaseless enquirer into scientific, technical and social studies, his concern for the development of children, teachers and teacher education is growing in importance. Educational ideas, like scientific probings, did not develop in a vacuum, and they have their principles 'rooted in the wisdom of the ages,' traced back to the ancient Greek thinkers like Socrates and Plato. Since then the ideas of many great thinkers, representing different disciplines, have been woven into the fabric of current educational philosophy and practice. Jean Piaget drew heavily from this great philosophical reservoir. While doing so, he appears to have paid back his ancestral debt abundantly in his own life time; indeed, it is impossible to imagine today a cognitive study without any reference to his work. Whether one agrees or not a dip in the 'Ganges' of Piget does affect one's outlook on educational problems. Why ? Because he presents a series of challenging syntheses as obtained in the rationalist tradition of Plato, mental categories of Kant, the Bergsonian notion of perceptual change, the findings of Gestalt psychologists like Max Wrtheimer, the use of logic for interpretation of thinking (classes, relations, grouping, reversibility and equilibrium), and in the revelatory statements made by personalities like Calvin, Rousseau, Pestalozzi, Claparede, Rinswanger, Bleuler. Rorschach, Meili, Jung and Sechehaye.

Philosophical problems present themselves to the philosophers, perhaps in the true tradition of their beloved field of enquiry in unsolvable form. Piaget, while still remaining a firm speculator, succeeded in psychologically tackling an age-old epistemological problem as enunciated by Plato :

'How and when can man be sure that his knowledge is true knowledge, when time and again he finds that what he took to be knowledge has proved to be error. In this, he not only substituted the fairly well-established scientific procedures with his own Methode clinique, but also constructed his own symbolic logic for interpreting intellectual operations as they develop from the very early childhood to late adolescence. He thus showed how a child nay, a 'universal child' goes about the business of constructing his own house of knowledge. Continuously challenged by this problem over a period of forty years, he came up with this grand hypothesis :

> Development is continuous not only within the individual but throughout all evolutionary levels. From the biological through the social to the intellectual levels, the unity of nature is preserved. The functioning of the loveliest mollusk is based upon the same fundamental processes as that of an Einstein.

The Nature of the Problem

Thinking is a 'ghost-like' activity, for it has to be inferred rather than observed. In literature, it has been referred to as 'abstracting, analysing, comparing, deducing, defining, discriminating, estimating, generalizing, guessing, imagining, judging, knowing, opining, reasoning, recalling, recognizing, reflecting, remembering, searching for conclusions and understanding.' While going ahead, one has to cross the territory of 'concept formation,' which again turns out to be a vast area of investigation in its own right, having strong links, but identical with such complex psychological processes as thinking, learning, problem-solving, language acquisition and symbolic representation. If this journey is further continued, one enters the complex areas of problem-solving, creativity, critical thinking and originality, where one is usually caught in a quicksand pit of definitional difficulties. One is very lucky, if for every miligram of information, one spends a quintal of effort.

And at this point, the distinction between original sense and original non-sense disappears.

Not long ago, the nuclear scientist Oppenheimer aptly remarked: 'It is the business of science to go wrong.' Difficulties for the cognitive psychologists, therefore, go on multiplying, and for achieving success, problems have to be posed more and more productively in the phraseology of Gestalt psychology with a view to investigating the complex processes of thinking, which may ultimately head to the development of understanding, generalization, discrimination, concept development and attainment. The problems, however, do not end here, because the researchers and the practitioners do not invariably ask the same question. These split-questions, and possibly the consequent split answers, are bound to appear in the business of any science, educational psychology being no exception. The workable solution under these circumstances lies in considering the two in isolation as is frequently done in meeting the anomalies in science. The basic problem here is to relate the most significant concepts of science to the mental development of children whose answer goes a long way in providing the psychological structure to any one of the school subjects. Thanks to the recent efforts of Ausubel, Bartlett, Beard, Bruner, Flavell, Gagne, Guilford, Hansfurth, Hearnshaw, Humphrey, Inhelder, Karplus, Lovell, Lunzer, Peel, Piaget, Schwab, Skinner, Suchman, Vernon, and Wallace, the problem-territory in this decade has been sufficiently explored and consequently, the problems in pieces—like the discovery of fundamental particles in atomic physics—lie more in the zoo rather than in the jungle.

This was hardly the case over twenty-five years ago. The emerging literature on thinking clearly tells us that children learn spontaneously, and they require schemes of

thought regardless of school influences. They learn from each other, and learn equally well in formal situations. At the same time, they also possess first hand knowledge about men, and their affairs in the immediate environment. This, however, does not amount to saying that the sun rises in the east or sets in the west. It only means that the child's thinking is not at all a chance or random behaviour. It is sensible, intelligible and predictable against the available theoretical psychological constructs.

Analysis of Thought

While delving deep into the Piagetian thinking, it is necessary to understand the contextual meanings of the concept of operations. Professor Bridgman in U.S.A was the first to champion that operationalism provides a real meeting ground for psychology and logic. As is known, operations play an important part in logic which is based on abstract algebra and is made up of symbolic manipulations. Here, operations are considered real psychological activities on which our whole effective knowledge is based. Piaget has, therefore, attempted to develop a psychological theory of operations which links psychology to logic. Roughly speaking, operation is a means for mentally transforming data about the real world so that they can be later on organized and used selectively in problem solving. Operation is internalized and reversible and, thus, distinguishes itself from a simple action or goal-directed behaviour. Consider the following definition of operations as given by Piaget :

> Psychologically, operations are actions which are internalizable, reversible, and co-ordinated into system characterized by laws which apply to the system as a whole. They are actions since they are carried out on objects before being performed on

symbols. They are internalizable since they can also be carried out in thought without losing their original character of actions. They are reversible as against simple actions which are irreversible. In this way, the operation of combining can be inverted immediately into the operation of dissociation ... Finally, since operations do not exist in isolation, they are connected in the form of structured wholes.

These operations evolve gradually in stages Whereas the age ranges may vary from sample to sample the sequence of development of these stages remains invarient, that is, does not change. As far as the current evidence goes, no intermediate stage is seen to have been skipped by anyone. Being a developmental psychologist immersed fully in epistimology, the four well-known stages following each other as enunciated by Piaget are given below in the tabular form :

Stages and their Characterlstics

S. No.	Stage	Age	Brief characteristics & abilities
	Sensory motor period	0-2 yrs.	From the educational point of view, this stage is not much of educational significance. —Child is highly dependent upon his parents for satisfaction of his physical needs —Performs overt activities —He appears to throw the whole of himself physically into the activity. —He thinks least about his actions. – He is attracted by sound, touch and other physical stimuli and while crawling he is on a voyage of discovery. —Flat pictures do not attract his attention but if he is given a pencil, he can stretch his hand to grasp it.

5

2.	Pre-operation stage	2-7 yrs.	—He sees things in his own frame of reference. —He can think only of one idea at a time. —He uses language in a limited way. Why ? Because he feels difficulty in expressing his thoughts. —He can carry on conservation with himself and in this he least cares whether others listen to him or not. —His social interest begins to widen and he tends to become less and less self centred. —His thinking is governed by the perceptual considerations of the situation. His reasoning is then said to be transductive.
3.	Concrete operational period.	7-11 years	—Child's thinking is tied to the concrete situations. He cannot react to abstract situations. —He just begins to attack problems systematically. At the same time, he cannot find mathematical proofs. —He does not accept hypothetical data. —Reality dominates his thinking and the possibility is subordinated to it.
4.	Formal operational period	11-15 years	—His thinking is no longer tied to the concrete situations. He imagines and considers all sorts of facts, hypotheses & possibilities. —He develops the ability to reason by hypotheses. He goes even to the extent of finding empirical and mathematical proofs for his observations.

The Geneva school considers the following five transformations which somehow inform us the way the concrete structures available at the third stage change into formal structures. These are :

1. The adolescent pupil is in a position to state as well as test hypotheses. This type of reasoning is called the hypothetico-deductive one.

2. The adolescent pupil is able to make the effective use of propositional logic.

3. The adolescent pupil is in a position to separate form from content. Here possibility rather than reality becomes the chief distinguishing characteristic of his thought.

4. The adolescent pupil is able to deal effectively with the entire combinatorial nature of operations that is, from the 16 binary combinations to 256 tertiary operations. It is a form of closely knit system in which 'passing from one element of structure to another is always possible.'

5. The adolescent is in a position to generate all the possible cases which are derivable from one single identifiable mental structure i. e. the INRC Group where the scripts have their usual meanings, that is, I (identity); N (negation); R (reciprocity), and C (correlative or dual operation) It is a fascinating mental structure, arising from the fusion of two different structures' namely, the lattice and the group of reversibilities.

Recently, Jean Piaget has also hypothesized the existence of the fifth stage as well which he attributes to an aptitude variation, specialization and also commitment to a particular career.

The Genevan Method
of
Attacking Problems

Piaget does not use standardised procedures (perception excluded). He chooses methods and techniques reflectively for effective explanation relative to the phenomenon under study. He develops his own concepts which are at times misunderstood, misquoted and found to suffer not only from defects but also he is found to apply them differently i. e., their meanings appear to change depending upon the period of his work. Difficulties of the readers increase when he does not at all explain his concepts sufficiently well but takes them for granted in his abstract reasoning. At the same time, while sticking to his field, he evaluates critically the findings of others and accepts criticism in turn.

Piaget's appoach is elastic and flexible. His method is clinical which demands both imagination and critical sense for great care is taken in not imposing the experimenter's point of view on the child Further, Piaget is able to establish a very good rapport with his subjects. In one of his experiments, he proceeded somewhat as follows :

Piaget : What am I doing ?
Pupil : You are clapping ?
Piaget : What am I doing ?
Pupil : You are slapping

Then Piaget presents the problem. He thus succeeds very well in giving us the inside view of what happens when children think. He does this by observing his subjects in informal situations, that is, under natural conditions, questioning them verbally about tneir everyday concepts round the natural phenomena; and investigates their thinking through his highly ingenious experimental materials and physical experiments. He then classifies their responses

into stages and discusses the main distinguishing features of each stage; and how it is linked with both its predecessor and successor stages. He also reports analysis for frequencies of responses and dispersion by ages. He then links these stages to his highly mathematical and abstract logical models. He hardly computes the conventional sort of statistical measures. He hardly uses tests of significance. This naturally publicizes a false impression that Piaget does not have any grounding in statistics and mathematics. To such critics, he has given one apt reply :

> The object of these studies, initially was not to establish a scale of development and to obtain precise determinations of age as regards stages. It was a question of trying to understand the intellectual mechanism used in the solution of problems and of determining the mechanism of reasoning. For that we used a method which is not standardized, a clinical method. a method of free conversation with the child..... That is why, personally I am always very suspicious of statistics on our results. Not that I dislike statistics, I worked on biometrics enthusiastically when I was a zoologist, but to make statistical tables on children when each was questioned differently appears to me very much open to criticism as regards the results of the dispersion.

He has his own reasons for selecting and developing these unconventional techniques. One should not miss the point that Piaget aims at investigating and finding out as much as he can (may it be at its worst considered a sort of exploration) about successive cognitive structures in the whole process of ontogenetic development among normal children. Therefore, researchers on Piaget have simply to learn to live with them (techniques) for the essence of his critical methods is to separate the wheat from the tares and keep every answer in its mental context, the 'context

may be one of reflection or of spontaneous belief, of play, or of prattle, of effort and interest, or of fatigue. (Lunzer).

It is just recently that Piaget has expounded his eclectic method of research and theory construction. In these, he employs two essential approaches which have much in common with those applied to nuclear sciences. These are a detailed analysis, in order, of an investigation based upon cause and effect constituting a net-work characterized by hierarchical relationships and combined connections and the consequent analysis of implications by considering both the field as a whole and the co-ordination of its parts in mathematics, this is comparable to the group, and in logic, to the propositional operation. Empirical investigations and the inductive mode of reasoning by logic and constructing hypotheses by deduction is similar to be second. Piaget believes and stresses that symbolic logic like statistical techniques is a productive research tool in psychology as well. One of his colleagues, Inhelder says :

> The use of such tools in no way implies that the psychologist has succumbed to logicism, that is, has decided in advance that the real thought of the child should conform to the laws which govern logical and mathematical structures. Only the facts can decide whether or not it does so conform..... These models represent the ideal system of all possible operations, while actual thought makes but one choice amongst them. More than twenty years of research have shown that cognitive development approximates these models without attaining them completely.

THE SCHEMES OF THOUGHT

It is necessary to understand the genesis of this term as used in psychological literature as well as the meaning given in this study with special reference to science. In British psychology, R. C. Oldfield and O. L. Zangwill analysed the meaning of this term in their paper on 'Head's concept of schema' and its application in contemporary British psychology about thirty years ago. Head suggested this term while studying 'control of bodily movement and localisation of tactile stimulation'. For example, a man is able to scratch his left or right ear from several diverse starting positions. He, therefore, hypothesized the existence of Image which a man carried in his head which exactly corresponded with its physical counterparts, a case of Kinaesthetic perception. F. C. Bartlett, while giving similar treatment to memory, took his cue from response and stated that skilled behaviour is eventually schematic in character. To put in other words, it meant that scheme not only determines any part of the response but also the whole sequence of responses which could be flexible under certain conditions. Piaget gave this term a comprehensive meaning and rooted it in biological behaviour (scheme).

Let us consider scheme first. It is the same thing as schema, plural being schemata. If loosely considered and understood, it could cover several terms like 'strategy, concept, plan, decision process, orientation, sequence of links, structure of hierarchical nature, learning set, cognitive structure, determining tendency, set of equivalences, search model or strategy, awareness of total layout, utilization of logical rules, and key factor to a varying degree each having past, present and future. To strengthen this consideration further, a schema at any moment can be considered to be composed of sub-schema, each sub-schema then being also called schema with mutual

implication of varying inter-connectedness at that very moment of development. At the time of its origin, it encompasses strategy and, in general, reflects relations among strategies at every level. So it is easier to comprehend that a schema comprises strategies or sub-schemata, a few of them, at least, active during problem solving. It has the inherent potentiality to organise itself at different levels. If not available at a particular moment, an inferior one may become active and seek accommodation to the problematic situation fully.

It is then not difficult to see that the various schemata, as they develop, are related to one another with the inherent possibility of the superior one controlling their inferior allies. Only when seen thus, a psychological organ parallel to physiological one is not only contemplated but also created on a very simple and plausible principle: all behaviour is related or essentially schematic in character without even suffering any loss of individual contextual meanings of the various schemata. So schema is nothing but the sort of algebraic generalisation or even individualised procedures of the available strategies taken individually in groups having content and form which disciplines the inside. If this workable view of schema is accepted, then it is not difficult to state the following attributes on functions of schemata as propounded by Piaget.

(a) It is as simple as reflective activity and as complex as advanced problem solving plan. It thus grows and organises itself at different levels, both horizontal and vertical.

(b) It is invisible that is, not open to direct observation. It is, therefore, inferred only.

(c) It assimilates as well as accommodates to external reality. Over the years, it is raised to the concept of a psychological organ having its parallel physiological counterpart.

(d) It does not develop in a vacuum. It is also not empty from inside. It is active as well as flexible having both content and form. The latter disciplines the former, for example, differentiating. In summary, the internal structure and the differentiated functions among the various components of the scheme are mutually inter-related, even compensation being no exception.

(e) In case of failure, another schema may become active for it is in the nature of scheme to act on a situation: physical or mental. This implies the existence of possible relationship among schemata. The very discrepant situation causes strain in the organism, which meets it by the very growth of schema sooner of later. Otherwise, it shows its coordinating functions with greater possibilities for the transfer of generalized search strategies.

(f) Lastly, it reflects itself by a given behaviour sequence or in it, the very utilization of logical rules. These are, comparatively speaking, easy to determine from the very knowledge which is being investigated upon from the angle of psychological formation. It is, thus, easy to see that the schema becomes the set of equivalence implied by the behaviour of the subject not, of course, in any single sequence, for no equivalence could be inferred from any isolated observation.

It is therefore, least surprising that their phychological investigation is a very exciting problem which has to be seen against two additional basic standpoints of the Geneva school. The first is that complex thinking processes arise or stem from simple thinking processes. Secondly, intellectual development (or thinking) takes place in stages. This adds its own difficulties to the psychometric treatment of the Piagetian concepts.

Having said this, very little is known about the mathematical structure of formal thought as visualized by Piaget. This publication, therefore, presents several researches done in India, U. K. and U. S. A. within the last two decades or so.

CHAPTER 2

The Mathematical Structure of Formal thought

Introduction

It is no exaggeration to point out that immense opportunities of research lie ahead for Piagetian workers using psychometric methods. In this frame of reference, very little is known about formal thought because it is yet to be delineated, understood, described and tested. According to R. L. Kimball, Piaget goes on high speeds ahead and leaves too much for speculation of the investigators to tackle when it comes to drawing inferences about definite hypothetical models and structures. On reflection, it just appears to be the growing process or formulating hypothetic-deductive models and validity testing hypotheses contained in those models for attaining predictive validity in experimental findings. Below are now consolidated a few factor analytic studies trying hard to illumine the Piagetian formal thought.

Using this approach, Thurstone in 1938 distinguished three types of reasoning ability; inductive reasoning, deductive reasoning, and restricted reasoning which have repeatedly appeared in other factorial studies. Guilford has further sub divided these into sub-abilities. Burt has identified three components of the reasoning ability as differentiated at adolescence, these being induction (education of conceptual relations, perceptual relations, and structural relations); deduction (education of correlates, structural correlates; and logical reasoning) and restrictive one.

Guilford and his associates have then attacked the phenomena of problem solving factorially. Steps in problem solving were determined by logical analysis. As all

problems are not of the same type, they further considered problems with reference to content (figural, symbolic, semantic, behavioural); operations needed to solve problems (preparation, analysis, production verification and reapplication) and products or outcomes (units, classes relations, systems, transformations and implications). They found that problem solving itself is as vast as behaviour itself. Their studies also indicated a key factor quite similar to strategies (Goodnow and Austin 1956) and Search Model (Dunker1945) which determines the efficiency (effectiveness) of creative activity or thinking. It is also bound to be of interest to consider another model for experimental work as suggested by Prof. J. Eysenck.

Peel (1955) surveyed factorial studies in the teaching of science in England and concluded that there does not appear to be a 'group factor' in science. On the other hand there appears a 'complex of general intelligence, reasoning, spatial number' and other factors. In another interesting study by Abou Hatab, on the definition and measurement by verbal methods of the ability to think critically, it was seen that :

> The hypothesized factors of verbal critical thinking were not isolated as independent of other intellectual factors in a sample of 170 Grammar school, sixth formers of the age 17 years; The main factors appeared, each includes various cognitive, productive and evaluative functions and requires different intellectual ability, namely, analytic thinking and intuitive thinking. Critical thinking factors did not appear because these appeared to depend largely on general intelligence, reasoning and verbal ability.

There is a general agreement, however, that formal thought depends upon concrete thought so far as its emergence concerns combinativity, reversibility, associativity

and identity etc. available at concrete stage. This possibly explains the emergence of operational schemata like combinatorial, proportional, mechanical equilibria, correlational and probabilistic etc. at the beginning of the fourth stage. The research point being made is that the various operational schemata have not only their own individualistic hierarchical structures but also possess the same across them as well. The studies of Lunzer and Pumphrey (1966); Lovell and Butterworth (1966) and Abramowitz (1975) show that formal operational thought is pre-requisite to the solution of problems involving proportional relationships. If this view is accepted, the same thing applies to schemes of thought which, in other words, means that in the absence of a particular full fledged scheme of thought, a part of it comes into play automatically. In analogical terms, it is like the molecular theory of magnetism. W.M. Bart found that formal thought had a bifactor structure. In it, it had both the verbal and non-verbal components of intelligence. It does not help much if formal thought or reasoning is regarded as a potential competency or (it is assumed) as the availability of formal thought very necessary in the acquisition of abstract concepts. R. O. Guerin here goes a step further when he investigates formal thought with the help of Raven's test of logical operations, using Guttman's simplex Analysis, as well as Kailer's alpha factors models' and independent cluster transformations, and concludes that logical operations of classification, seriation, logical multiplication, compensation, proportional thinking, ordinality, and correlational thinking do not at all show hierarchical character. According to him, formal thought or logical operations have two factors, namely, 'grouping, concrete operational and co-ordinating concrete logical'. He further adds that the logical operation of compensation is related to both factors and represents a transition operation between the purely concrete operations and the purely logical or formal operations.'

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THE PRESENTATION OF STUDIES

1. In 1964, N. Vaidya investigated adolescent thought, particularly speaking, problem solving, with the help of nine problems selected from physics in varied Piagetian contexts. The problems related to schemes of summation, constant difference, conservation, proportion, categorization, stating and testing of hypotheses and discovery of principles from incompletely supplied data. This study was conducted in two parts, that is, using a questionnaire approach (N = 60) and an individual approach (N = 31) The sample for the study was drawn from a secondary modern and a comprehensive school in Central London. Other variables used in this study were : intelligence, age, 11 + test scores on Arithmetic and English, persistence, maladjustment and interest in things. Using Hotelling method, the following mathematical structures of the tasks used appeared in his study. Out of the ten factors extracted, only four were considered significant for interpreting them from the psychological point of view. Two factors were rotated at a time. Rotation between Io & IVo appeared to be the best one in the sense that it showed a good cluster. Out of the many rotations made, the following rotations were selected finally.

TABLE No. 1

Showing Rotating Factors, Angles of Rotation and the Resulting Factors.

S. No.	Rotating factors	Angle of rotation	Resulting factors
1.	I_0 and IV_0	6°	I_1 and IV_1
2.	I_1 and II_0	19°	I_2 and II_1
3.	I_2 and III	19°	I_3 and III_1

TABLE No. 2.

Showing the Values for the Factors on the Variables After Rotation.*

S. No	Variables	Description	I_3	II_1	III_1	IV_1
1.	I. Q.	On 11+test	.483	—	.668	—
2.	Age	15+ test	—	—	—	—
3.	Arithmetic	11+test	.647	—	—	—
4.	English	11+test	.575	—	.576	—
5.	Self-confidence	Personality test	—	−.368	.348	.477
6.	Persistence	Personality test	—	—	.366	.691
7.	Maladjustment	Situations that worry & frighten people	—	.666	—	−.496
8.	Interest in things	Link camping, news-papers & puzzles etc.	—	—	.579	—
9.	Flask problem	Schema of conservation & proportion	.654	.436	—	.456
10.	Beaker problem	Possible hypothesis setting up	.615	—	—	—
11.	Fish problem	Schema of proportion & conservation	.546	.387	—	—
12.	Spring balance	Categorization & proportion	.571	.448	—	—
13.	Lens problem	Summation, proportion & application	.537	.462	—	—
14.	Electric circuit problem	Hypothesis formation	—	.470	—	—
15	Refracture index problem	Proportion	.785	—	—	—
16.	Two Temp. scale problem	Constant difference	.721	—	—	—
17.	Simple pendulum problem	Setting up control experiments	.393	.682	—	—

* Factor loadings below ±.35 are not shown for all similar studies included for consideration here.

The factors were then interpreted as follows :

(a) Attainment factor (I_1). Intelligence, artithmetic, English and all the problems have positive as well as high loadings on this factor. Selfconfidence, maladjustment. persistence, and interest in things have low and insignificant loadings on this factor. This factor can, therefore, be interpreted as an attainment factor.

(b) Practical factor (II_1). Flask problem, Spring balance problem, ens problem, Electric circuit problem and Simple pendulm problem have high positive loadings on this factor. These problems do demand some sort of mechanical experience or they are in a way more of a practical nature. This is also supported by the fact that English, arithmetic and even intelligence have very low (and largely insignificant) loadings on this factor where-as personality variables have high loadings on this factor. This factor can therefore be interpreted as either mechanical experience or practical factor.

(c) Interest factor (III_1). Next to intelligence, interest in things has the highest positive loading on this factor. All the problems mostly have insignificant loadings on this factor. This factor can therefore be interpreted as interest factor.

(d) Adjustment factor (IV_1). Self confidence, persistence and adjustment have high loadings on this factor. The rest of the variables i. e., English, arithmetic, intelligence and most of the problems have insignificant loadings on this factor. This factor can therefore be interpreted as adjustment factor.

The above mentioned study suffered from several dificiencies : Study population was too small; the pupil age was highly restricted; and lastly, the problems used despite their number were not as diverse as expected in Piagetian studies. For interpretation of factors, factorially known tests were also not used.

2. In another study by the same author in 1975, another attempt was made to investigate Some Aspects of Adolescent Thought with the help of specially designed problems, each inhering a continuous chain of reasoning. This study, using the case-study approach, also attempted to delineate the structure of the various hypothesized schemes of Thought factorially in which the scores on individual problems were also used as additional variables, thus, pitting hypothesized schemes of thought drawn from the internal structure of problems themselves empirically and the individual problems each taken in isolation, against each other. Subsidiary findings being numerous, the main findings of this study indicated :

(i) Except occasional fluctuations, average performance on each problem increases with grade. Mean performance in most of the cases favours boys rather than girls. But both boys and girls try hard to equalise their performance as they move into higher grades.

(ii) A given problem, or a part of the problem, or a process in their problem, is solved successfully (or failed as well) over a wide I. Q. range both within and across the various grades.

(iii) A given problem is solved in stages. It is possible to identify stages in the solution of any problem.

(iv) Unexpectedly, pupils commit a large number of errors while engaged in problem-solving. These errors further increase in the higher age-groups, when they fail to grasp the main requirement of the problem. Among these errors, the dominent ones, shared by more than 15 per cent of the pupils largely speaking, suffer 'a hump' before they finally decline with increasing grade.

(v) Contrary to Piaget, pupils are not in a position to exhaust or suggest all possibilities, combinations and tests. Over three-fifths of the pupils from Grades VI

to VIII are badly affected when a problem inheres reversibility in thinking.

(vi) The complex problem-solving processes arise from simple thinking processes.

(vii) The role of the nature of the problem being critical in such studies on thinking, the adolescent pupils, contrary to Piaget, are affected to a great extent by the content of the problem.

(viii) Although the adolescent pupils are in a position to set up hypotheses, they are not in a position, contrary to Piaget, to test them which shows that their minds have not yet become experimental. Most of the adolescent pupils do not, contrary to Piaget, possess schemes of proportion as well as generalization to algebraic symbols.

(ix) When a problem is solvable through two schemes of thought, one inferior and the other superior, and if the latter is not well developed, the resort to the former may favour quite a few in solving that problem. In case both are conspicuous by their absence, there is little chance for the supplied hint or illustration or even analogy to be utilized in solving the problem successfully.

(x) Except occasional fluctuations, the mean performance on the various schemes of thought shows an increasing trend with grade. When mean and standard deviation are kept at 20 and 3 respectively, the gains in term of T scores over the five year period are dismal, implying thereby that they are slow and laborious as they evolve across the grades.

Investigating forty-five variables in all (17 problems, 17 schemes of thought derived across problems, grade, intelligence and a few other measures of adjustment), the factor loadings for the ten-factors on each variable are given in the tabular form given below. Then follows immediately the interpretation of factors.

TABLE No. 3.

Showing Loadings of Variables on Rotated Components After Rotation by Varimax Method* Boys & Girls (N=200)

S. No.	Variables	ROTATED COMPONENTS									
		I	II	III	IV	V	VI	VII	VIII	IX	X
1.	Length of schooling	0.74	—	—	—	—	—	—	—	—	—
2.	Intelligence	—	—	—	—	—	—	—	—	—	−0.83
3.	Home adjustment	—	−0.68	—	—	—	—	—	—	—	—
4.	Health adjustment	—	−0.65	—	—	—	—	—	—	—	—
5.	Social adjustment	—	−0.76	—	—	—	—	—	—	—	—
6.	Emotional adjustment	—	−0.72	—	—	—	—	—	—	—	—
7.	School adjustment	—	−0.71	—	—	—	—	—	—	—	—
8.	Understanding of the problem	—	—	−0.82	—	—	—	—	—	—	—
9.	Difficulty of the problem	—	—	—	—	—	—	—	—	0.50	—
10.	Confidence in the problem	—	—	−0.76	—	—	—	—	—	—	—
11.	Interest in the problem	—	—	−0.76	—	—	—	—	—	—	—
12.	Height problem	0.47	—	—	—	—	—	—	—	−0.48	—
13.	Positive summation series	—	—	—	—	—	—	0.80	—	—	—
14.	Negative summation series	0.48	—	—	—	—	—	0.41	—	−0.36	—

*Factor loadings below .35 are not shown.

S. No.	Variables	ROTATED COMPONENTS									
		I	II	III	IV	V	VI	VII	VIII	IX	X
15.	Proportion problem	0.41	—	—	—	−0.75	—	—	—	—	—
16.	Hotel problem	0.49	—	—	—	—	—	−0.55	—	—	—
17.	Rectangular problem	0.49	—	—	—	—	—	—	—	—	—
18.	Rectangular cubes problem	0.41	—	—	—	−0.38	—	—	—	—	−0.37
19.	Counting rectangles maximally	—	—	—	—	−0.48	—	—	−0.37	—	—
20.	Combinatorial digital problem	0.45	—	—	—	—	—	—	−0.61	—	—
21.	Questions involving wrong answers	0.69	—	—	—	—	—	—	—	—	—
22.	Nine dots problem	0.50	—	—	—	−0.52	—	—	—	−0.43	—
23.	Formulating questions problem	—	—	—	−0.91	—	—	—	—	—	—
24.	Beaker problem	0.35	—	—	—	−0.67	—	—	—	—	−0.40
25.	Fish problem	0.78	—	—	—	—	—	—	—	—	—
26.	Spring balance problem	0.68	—	—	—	−0.39	—	—	—	—	—
27.	Proposing tests problem	0.75	—	—	—	—	—	—	—	—	—
28.	Flow of water problem	—	—	—	—	—	−0.92	—	—	—	—
29.	Using constant difference	0.60	—	—	—	—	—	0.47	—	—	—
30.	Using summation	0.85	—	—	—	—	—	—	—	—	—

S. No.	Variables	ROTATED COMPONENTS									
		I	II	III	IV	V	VI	VII	VIII	IX	X
31.	Using proportion	0.80	—	—	—	—	—	—	—	—	—
32.	Beaker combination	0.35	—	—	—	−0.67	—	—	—	—	0.40
33.	Using two digits at a time	—	—	—	—	—	—	—	−0.69	—	—
34.	Using three digits at a time	—	—	—	—	—	—	—	−0.67	—	—
35.	Using four digits at a time	—	—	—	—	—	—	—	—	−0.62	—
36.	Generalization to algebraic symbols: summation	—	—	—	—	−0.39	—	0.58	—	—	—
37.	Generalization to algebraic symbols : proportion	0.42	—	—	—	−0.75	—	—	—	—	—
38.	Stating hypotheses	—	—	—	—	—	—	—	—	−0.45	—
39.	Testing hypotheses	—	—	—	—	—	−0.92	—	—	—	—
40.	Stating procedures	0.65	—	—	—	−0.35	−0.49	—	—	—	—
41.	Proposing tests	0.75	—	—	—	—	—	—	—	—	—
42.	Formulating problematic situations (fluency)	—	—	—	−0.92	—	—	—	—	—	—
43.	Formulating problematic situations (flexibility)	—	—	—	−0.76	—	—	—	—	—	—
44.	Using insight	0.45	—	—	—	−0.63	—	—	—	—	—
45.	Failure to grasp the essence of the problem	0.69	—	—	—	—	—	—	—	—	—

TABLE No. 4

Showing Latent Roots as Well as Percentage of Trace Total Variation of Each Component and Accumulated Per Centage for the Ten Factors (Boys and Girls. N=200).

S. No.	Factors	Latent roots	Percentage of trace total variation accounted	
			Each cemponent	Accumulated percentage
1.	Schematic Learning Gen.	15.93	35.40	35.40
2.	Adjustment	3.05	6.77	42.18
3.	Problem orientation	2.25	5.00	47.19
4.	Sensing problems	2.22	4.94	52.13
5.	Symbolization	1.94	4.30	56.43
6.	Testing hypotheses	1.72	3.82	60.25
7.	Using constant difference	1.57	3.49	63.75
8.	Aspect character	1.49	3.33	67.07
9.	Seeing the problem as a whole	1.36	3.02	70.10
10.	Intelligence	1.19	2.65	72.75

Interpretation of Factors

First factor : It has ahigh positive loading on 25 variables. Except the factor loading of school adjustment on this factor (.279 which is low), all other factor loadings relating to the various aspects of adjustment and immediate test reactions to the problems of presentation are insignificant. Further, the following five variables have insignificant loadings on this factor :

1. Generalisation to algebraic symbols (Summation) — =0.560
2. Stating hypotheses =.1420
3. Testing hypotheses =.2030
4. Formulating problematic situations : Fluency =.1908
5. „ „ „ : Flexibility =.1580

There is another interesting observation as well. Unexpectedly, the factor loading of Intelligence on the first factor, of course, positive is found to be insignificant (0.1678). On the other hand, the loading of another variable, i.e., Grade is found to be on the high side (0.7382). Seen concretely, this variable is tied closely to the secondary school years implying thereby that the various schemes of thought (twelve out of seventeen) are strongly dependent on it. And they emerge during secondary school years. It should not be lost sight of, that problems used in this study were solved gradually short of full completion in the closing grades only. Under the circumstances, when factors are defined more by the inside qualitative analysis rather than by external features (form and material) as well as when intelligence is hardly found to differentiate during adolescence, secondary education ought to foster integration rather than differentiation of verbal as well as practical abilities in India (by implication) as in Britain (according to Professor P. E. Vernon). It is a broad hypothesis which needs additional exploration through several factorial studies. In this context, there is then no alternative left except naming the first factor as Schematic Learning General appearing during Secondary school years. When a search for this type of factor is made in literature, it comes very close to complexity as propounded by R. Meili which means 'ability to realise complex intellectual structures.' In his interpretation, he rejected g, V, N, or S because these arose from the very consideration of the external characteristics of the tests included in various factorial studies by the past workers. P. E Vernon, however, regards these factors of doubtful value, reason being his psychometric rather than developmental orientation towards the analysis of intelligence in the writer's opinion. While investigating the growth of the Schemata of Proportion during adolescence, K. Lovell

extracted a general component 'accounting for a little over 44 percent of the variance' and termed it Central Intellectual Ability which encompassed the said 'schema.' He further stated that the development of this scheme is slow during the secondary years, a finding which is consistent with the finding of this study as well. But the Central Intellectual Factor is quite different from Schematic Learning General because the former is tied to general educational ability in the British tradition rather than the several evolving schemes of thought linked strongly with grade but little with intelligence as hypothesised in this study.

Second factor : Variables only relating to Adjustment have high loadings on this factor. The factor of 'Felt difficulty of the problem', can be ignored (.2870) Only the problem (Rectangular cube problem) and one scheme of thought (Using two digits at a time) have significant loadings on the factor, the former being −.2765 (which can be ignored) and the latter −.3214 which is slightly above .30, the highest size so far usually ignored in the factorial studies. So this factor can be interpreted as adjustment factor for the five tests reflecting this main variable have pretty high loadings on this factor. Their sizes are :

1. Social Adjustment	= −.7619
2. Emotional Adjustment	= −.7180
3. School Adjustment	= −.7127
4. Home Adjustment	= −.6765
5. Health Adjustment	= −.6482

Third factor: This factor appears very clearly and dominantly and is closely akin to attitude towards 'Problem Solving' or Problem Orientation. Intelligence, grade, adjustment and all problems as well as schemes of thought have significant loadings on this factor. The factor loading of the 'Felt difficulty' variable is positive because it was scored in the reverse order. Secondly, its size is quite

small and hence can be ignored (.2841). Other tests having significant loadings on this factor are :

1. Understanding the problem = −.8217
2. Interest in the problem = −.7614
3. Confidence in the problem = −.7593

This factor is, therefore, interpreted as Problem Orientation.

Fourth factor : This factor also appears quite clearly on which grade, intelligence, sixteen problems and fourteen schemes of thought, out of seventeen, each have significant loadings. The two main tests or variables having significant loadings on this factor demand, on the part of the pupils, formulation of those questions whose answers they do not know as well as grasping the essence of the problem. It may be added that the type of question asked determined its category, that is, flexibility in contrast to fluency in this study. On further reflection, this factor in one form or other is closely related to creativity for it is being determined, namely, not only by the open problem but also by a series of contextual questions, whose answers little depend upon arithmetical rules learnt mechanically for they intentionally attract erratic thinking. Here, reference is to another Problem in which several questions intentionally invite wrong answers. It is also worth noticing that all tests or variables relating to problem orientation factor (already identified) and adjustment have significant loadings on this factor. The tests or variables which have highly significant loadings on this factor are :

1. Formulating problematic situations = −.9144
2. Formulating problematic situations (Fluency) = −.9152
3. Formulating problematic situations (Flexibility) = −.7589
4. Grasping the essence of the problem = −.3378

Earlier, R. Meili, using similar approach but different tests and subjects, extracted two factors bordering on creativity which resemble the present factor under inter-

pretation when seen from the angle of Central mental function underlying it :

1. Plasticity : the breaking down and reorganisation of structures. This seems to resemble Thurstone's second perceptual factor closely.

2. Fluency : This corresponds to Thurstone's unrestriced or ideational factor.

To reiterate, both the variables relating to stating hypotheses and testing hypotheses have insignificant loadings on this factor. This factor is, therefore, interpreted as Sensing Problems Factor which is closely identifiable with 'plasticity in thought processes', fluency, or interest in generating difficult problems (questions) yet in search of answers.

Fifth factor : It runs through the following variables: grade, seven problems and eight schemes of thought. Interestingly enough, it does not run through variables relating to both adjustment and problem orientation factors. It also does not run through two open problems : proposing tests and formulating problematic situations. The factor loading of grade on this factor (also the ninth factor yet to be interpreted) nearly reaches significance and hence has a say in the interpretation of this factor. It throws a broad hint that the factor under study or interpretation is closely tied to the length of the schooling which in this study is closely related to secondary education. In order to pinpoint sharply on this factor, consider below a few top loadings of six variables or tests on this factor :

1. Proportion problem = −.7478
2. Generalisation to algebraic symbols (Proportion) = −.7464
3. Beaker problem = −.6697
4. Beaker combination = −.6697
5. Using insight = −.6259
6. Nine dot problem = −.5237

These tests require two schemes of thought in particular: proportion and insight (structuring and restructuring). In the proportional schemes of thought, generalisation of arithmetical operations to algebraic symbols in proportion context is involved. Side support to this consideration is further provided by Rectangular Cube Problem (−.3776); and Spring Balance Problem (−.3910). Lastly, whereas the scheme of thought related to Testing Hypotheses has significant loading on this factor, the other two schemes of thought (stating hypotheses and proposing tests) have just approaching significant loadings on this factor. All these considerations give indication that this factor is yet incomplete, that is, one sharp remove or step away from general generalization of arithmetical operations to algebraic symbols but, at the same time, still inhering a very important scheme of thought, that is, using insight. It is therefore, interpreted as symbolisation which is closely identifiable with using Algebraic symbols in proportion context or short of it, Space visualisation (characterised by structuring and restructuring) factor as propounded by Thurstone.

Sixth factor : This factor also emerges clearly. It has quite a high loading on the Flow of Water problem (or the scheme of thought relating to testing and verification of hypotheses). Stating procedures (or verbalising methods of attack variable) has the second highest loading on this factor. The remaining fifteen schemes of thought as well as fourteen problems have insignificant loadings on this factor. The two problems still left (Rectangular problem and counting rectangles maximally) have loadings of the same sign which just approach significance. The significant loadings on this factor are shown below :

1. Flow of water problem = −.9174
2. Testing hypotheses = −.9174
3. Stating procedures = −.4861

4. Counting-rectangles maximally = −.3037
5. Rectangular problem = −.2520

The problem : Counting rectangles maximally involves considering and reconsidering the same problematic diagram from as many different aspects as possible. This factor is, therefore, interpreted as Testing hypotheses which closely resembles verification of hypotheses keeping in mind the fact that methods of attacking the problem were given verbally without actually carrying out the specific control experiments (Manipulation of experimental materials was allowed).

Seventh factor : This factor runs only through the problems and two schemes of thought. Particularly speaking, grade, intelligence, adjustment and immediate test reactions are the various variables which have insignificant loadings on this factor. Like the Fifth factor, it closely lacks symbolisation, i.e., handling arithmetical symbols in a context characterised fully by the overall generalisation of using constant difference to algebraic symbols. The following significant loadings of the five tests support the above interpretation :

1. Positive summation series problem = −.7983
2. Hotel problem = −.5487
3. Generalisation to algebraic symbols (Summation) = −.5812
4. Using constant difference = −.4720
5. Negative summation series problem = −.4101

It appears that this structure is still evolving in relation to its Generalisation to algebraic symbols. This factor is therefore, interpreted as Using constant Difference in arithmetical situations only.

Eighth factor : This factor runs through only four problems and three schemes of thought. Grade, intelligence and adjustment are the variables which possess insignificant loadings on this factor. When closely examined for

relevance, these seven problems and schemes of thought possessing significant loadings on this factor hint at a factor which prevents pupils from taking into consideration all aspects of the problematic situation simultaneously. To put concretely, they consider any one of the aspects of the situation at a time resulting in stayput in thinking so far as onward progress of the solution of the problem is concerned. In this context, the following loadings may be considered :

1. Using two digits at a time = −.6911
2. Using three digits at a time = −.6671
3. Combinational digital problem = −.6145
4. Using four digits at a time or seeing the digital problems as a whole = +.01258

The factor loading of the last test on this factor is very insiginificant. Therefore, this factor is interpreted 'as Aspect character' as propounded by Van de Geer in Phenomenology in his theory of problem solving and is closely indentifiable with either 'fixedness' in problem solving or with a condition in which pupils try hard to exhaust all the possibilities only in one direction at the cost of the second aspect.

Ninth factor : It runs through grade, felt difficulty of the problem, three problems and three schemes of thought. Intelligence, adjustment and immediate test reactions to the Problems on presentation are the tests which have insignificant loadings on this factor, exception being 'the felt difficulty of the problem.' In order of size, the factor loadings of the other tests which possess significant values of this factor are :

1. Using four digits at a time, or seeing the digital problem as a whole = −.6172
2. Felt difficulty of the problem = +.5002
3. Height problem = - .4783
4. Stating hypotheses = −.4536

5. Using constant difference = −.4263
6. Negative summation series problem = −.3659
7. Combinational digital problem = −.3392
8. Grade = −.3040

It is evident from the above that seeing the problem as a whole has the highest loading on this factor. Whereas the variables : 'Felt difficulty of the problem' has a moderately high loading on this factor (.5002). that of Grade tends to approach significance (−.3040). The latter gives indication that the factor is tied slightly to Grade (secondary education) depressed appreciably by the 'Felt difficulty of the problem.' When the eight and ninth factors are considered separately, hypotheses appear : before a problem is fully solved, it is first solved step by step with little concern or consideration of their order. Further, grade and felt difficulty of the problem influence the emergency of this scheme thought to some extent. This, factor is, therefore, interpreted, as 'Seeing the problem as a whole,' which is closely identifiable with consideration of all possibilities dominating thinking,' 'globalisation' : uniting separate data into a single whole, an essentially creative capacity (R. Meili) and 'seeing relations between and carrying out operations with numbers' (F. M. Earle).

Tenth factor : This factor runs through intelligence (−.8277) and two problems as well as two schemes of thought. The factor loadings of the latter are : −.2830 (height problem); −3698 (rectangular problem) and .3971 (beaker problem or combination. Further, variables relating to grade, adjustment and problem orientation have insignificant loadings on this factor. This factor, is therefore interpreted as Intelligence and has appeared for reasons not yet hypothesized by the writer.

Using part of the same data, it became possible to analyse the mathematical structure of adolescent girls only (N = 100). The data presented below suggests that there is a distinct difference in factorial structure of girls and those of the combined population of boys and girls.

TABLE No. 5.

Showing the Factors, Latent Roots of Correlation Matrix and Percentage of Trace Total Variation Accounted for Girls (N = 100)

S. No.	Factors	Latent roots of Correlation matrix	Percentage of trace total variation accounted	
			Each Component	Accumulated Percentage
1.	Schematic learning general	15.43	34.28	34.28
2.	Adjustment	03.72	08.26	42.54
3.	Problem sensitivity	02.95	06.55	49.08
4.	Tackling algebraic symbols	02.72	06.03	55.12
5.	Problem orientation	02.53	05.62	60.74
6.	Using constant difference	01.74	03.86	64.60
*7.	Aspect character	01.61	03.58	68.18
*8.	Stating hypotheses	01.37	03.04	71.21
*9.	Combinatorial grouping (digits).	01.34	02.97	74.19
*10.	Verbal description of procedures.	01.19	02.65	76.84

*Eliminated through Scree test.

TABLE No. 6.

Showing Ten Factors—Named for Boys and Girls (N = 200) and Girls (N = 100).

S. No.	Factors	Name of the factors.	
		Boys & Girls(N = 200)	Girls (N = 100)
1.	First factor	Schematic learning general	Schematic learning general
2.	Second factor	Adjustment	Adjustment
3.	Third factor	Problem Orientation	Problem sensitivity
4.	Fourth factor	Sensing problems	Tackling algebraic symbols
5.	Fifth factor	Symbolization	Problem orientation
6.	Sixth factor	Testing hypotheses	Using constant difference
7.	Seventh factor	Using constant difference	Aspect character
8.	Eighth factor	Aspect character	Stating hypotheses
9.	Ninth factor	Seeing the problem as a whole	Combination grouping (digits)
10.	Tenth factor	Intelligence	Verbal description of procedure

TABLE No. 7.

Showing Loadings of Variables on Rotated Components After Rotation by Varimax Method* Girls (N=100)

S. No.	Variables	ROTATED COMPONENTS									
		I	II	III	IV	V	VI	VII	VIII	IX	X
1.	Length of schooling	0.63	—	—	−0.38	—	−0.44	—	—	—	—
2.	Intelligence	—	—	—	—	—	—	—	−0.48	—	—
3.	Home adjustment	—	−0.75	—	—	—	—	—	—	—	—
4.	Health adjustment	—	−0.56	—	—	—	—	—	—	—	—
5.	Social adjustment	—	−0.84	—	—	—	—	—	—	—	—
6.	Emotional adjustment	—	−0.70	—	—	—	—	—	—	—	—
7.	School adjustment	—	-0.78	—	—	—	—	—	—	—	—
8.	Understanding of the problem	—	—	—	—	−0.78	—	—	—	—	—
9.	Difficulty of the problem	—	—	—	—	0.41	—	0.36	—	—	—
10.	Confidence in the problem	—	—	—	—	−0.71	—	—	—	—	—
11.	Interest in the problem	—	—	—	—	−0.68	—	—	—	—	—
12.	Height problem	—	—	—	—	—	−0.54	—	—	—	—
13.	Positive summation series	—	—	—	—	—	-0.70	—	—	—	—
14.	Negative summation series	—	—	—	−0.48	—	−0.55	—	—	—	—

*Factor loadings below .35 are not shown.

S N.o.	Variables	ROTATED COMPONENTS									
		I	II	III	IV	V	VI	VII	VIII	IX	X
15.	Proportion problem	—	—	—	– 0.85	—	—	—	—	—	—
16.	Hotel problem	—	—	—	—	—	– 0.79	—	—	—	—
17.	Rectangular problem	—	—	—	—	—	—	– 0.68	—	—	—
18.	Rectangular cubes problem	—	—	—	– 0.49	—	—	—	—	—	—
19.	Counting rectangles maximally	—	—	—	– 0.46	—	—	—	—	0.50	—
20.	Combinatorial digital problem	0.38	—	—	—	—	– 0.40	—	—	– 0.56	—
21.	Questions involving wrong answers	—	—	– 0.57	—	—	– 0.43	—	—	—	—
22.	Nine dots problem	—	—	—	– 0.51	—	– 0.46	—	0.37	—	—
23.	Formulating questions problem	—	—	– 0.90	—	—	—	—	—	—	—
24.	Beaker problem	—	—	—	– 0.74	—	—	—	—	—	—
25.	Fish problem	—	—	—	—	—	—	—	—	—	0.80
26.	Spring balance problem	0.39	—	—	– 0.47	—	– 0.36	—	—	—	0.36
27.	Proposing tests problem	0.77	—	—	—	—	—	—	—	—	—
28.	Flow of water problem	—	—	—	– 0.38	—	—	—	—	—	0.75
29.	Using constant difference	—	—	—	—	—	– 0.79	—	—	—	—
30.	Using summation	0.42	—	—	—	—	– 0.54	—	—	—	0.52

S. No.	Variables	ROTATED COMPONENTS									
		I	II	III	IV	V	VI	VII	VIII	IX	X
31.	Using proportion	—	—	—	—	—	−0.59	—	—	—	0.55
32.	Beaker combination	—	—	—	−0.74	—	—	—	—	—	—
33.	Using two digits at a time	—	−0.39	—	—	—	—	—	—	−0.72	—
34.	Using three digits at a time	—	—	—	−0.44	—	—	—	—	−0.67	—
35.	Using four digits at a time	0.52	0.35	—	—	—	—	—	—	—	—
36.	Generalization to algebraic symbols: summation	—	—	—	−0.79	—	−0.44	—	—	—	—
37.	Generalization to algebraic symbols : proportion	—	—	—	−0.84	—	—	—	—	—	—
38.	Stating hypotheses	—	—	—	—	—	—	—	0.76	—	—
39.	Testing hypotheses	—	—	—	—	—	—	—	—	—	0.75
40.	Stating procedures	—	—	—	—	—	—	—	—	—	0.87
41.	Proposing tests	0.77	—	—	—	—	—	—	—	—	—
42.	Formulating problematic situations (fluency)	—	—	−0.90	—	—	—	—	—	—	—
43.	Formulating problematic situations (flexibility)	—	—	−0.75	—	—	—	—	—	—	—
44.	Using insight	—	—	—	—	—	−0.40	—	0.38	—	—
45.	Failure to grasp the essence of the problem	—	—	−0.57	−0.62	—	−0.43	—	—	—	

TABLE No. 8.

Showing the Presence of Factors for Boys & Girls (N=200) and Girls (N=100)

S. No.	Variables	ROTATED COMPONENTS																					
		I		II		III		IV		V		VI		VII		VIII		IX		X			
		B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G		
1.	Length of schooling	+	+	—	—	—	—	—	+	—	—	—	+	—	—	—	—	—	—	—	—		
2.	Intelligence	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	+	—	—	+	—		
3.	Home adjustment	—	—	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
4.	Health adjustment	—	—	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
5.	Social adjustment	—	—	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
6.	Emotional adjustment	—	—	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
7.	School adjustment	—	—	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
8.	Understanding of the problem	—	—	—	—	+	—	—	—	+	—	—	—	—	—	—	—	+	—	—	—		
9.	Difficulty of the problem	—	—	—	—	—	—	—	—	+	—	—	—	—	+	—	—	—	—	—	—		
10.	Confidence of the problem	—	—	—	—	+	—	—	—	+	—	—	—	—	—	—	—	—	—	—	—		
11.	Interest in the problem	—	—	—	—	+	—	—	—	+	—	—	—	—	—	—	—	—	—	—	—		
12.	Height problem	+	—	—	—	—	—	—	—	—	—	—	+	—	—	—	—	+	—	—	—		
13.	Positive summation series	—	—	—	—	—	—	—	—	—	—	—	+	+	—	—	—	—	—	—	—		
14.	Negative summation series	+	—	—	—	—	—	+	—	—	—	—	+	+	—	—	—	+	—	—	—		
15.	Proportion problem	+	—	—	—	—	—	+	+	—	—	—	—	—	—	—	—	—	—	—	—		

S. No.	Variables	ROTATED COMPONENTS																			
		I		II		III		IV		V		VI		VII		VIII		IX		X	
		B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G
16.	Hotel problem	+	—	—	—	—	—	—	—	—	—	—	+	+	—	—	—	—	—	—	—
17.	Rectangular problem	+	—	—	—	—	—	—	—	—	—	—	—	—	+	—	—	—	—	—	—
18.	Rectangular cubes problem	+	—	—	—	—	—	—	+	+	—	—	—	—	—	—	—	—	—	+	—
19.	Counting rectangles maximally problem	—	—	—	—	—	—	—	+	+	—	—	—	—	—	+	—	—	—	—	—
20.	Combinatorial digital problem	+	+	—	—	—	—	—	—	—	—	—	+	—	—	+	—	—	+	—	—
21.	Questions involving wrong answers	+	—	—	—	—	+	—	—	—	—	—	+	—	—	—	—	—	—	—	—
22.	Nine dots problem	+	—	—	—	—	—	—	+	+	—	—	+	—	—	—	+	+	—	—	—
23.	Formulating questions problem	—	—	—	—	—	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—
24.	Beaker problem	+	—	—	—	—	—	—	+	—	—	—	—	+	—	—	—	—	—	+	—
25.	Fish problem	+	—	—	—	—	—	—	—	+	—	—	—	—	—	—	—	—	—	—	+
26.	Spring balance problem	+	+	—	—	—	—	—	+	—	—	—	—	—	—	—	—	—	—	—	+
27.	Proposing tests problem	+	+	—	—	—	—	—	—	+	—	—	—	—	—	—	—	—	—	—	—
28.	Flow of water problem	—	—	—	—	—	—	—	+	—	—	—	—	—	—	—	—	—	—	—	+
29.	Using constant difference	+	—	—	—	—	—	—	—	—	—	—	+	—	—	—	—	—	—	—	—
30.	Using summation	+	+	—	—	—	—	—	—	—	—	—	+	—	—	—	—	—	—	—	+

S. No.	Variables	ROTATED COMPONENTS																					
		I		II		III		IV		V		VI		VII		VIII		IX		X			
		B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G	B&G	G		
31.	Using proportiion	+	—	—	—	—	—	—	—	—	—	—	+	—	—	—	—	—	-	-	+		
32.	Beaker combinations	+	—	—	—	—	—	—	+	+	—	—	—	—	—	—	—	—	-	+	-		
33.	Using two digits at a time	—	—	—	+	—	—	—	—	—	—	—	—	—	—	+	—	—	+	-	-		
34.	Using three digits at a time	—	—	—	—	—	—	—	+	—	—	—	—	—	—	+	—	—	+	-	-		
35.	Using four digits at a time	—	+	—	+	—	—	—	—	—	—	—	—	—	—	—	—	+	-	-	-		
36.	Generalization to alg. symbols	—	—	—	—	—	—	+	+	—	—	+	—	-	—	—	—	—	-	-	-		
37.	Generalization to algebraic symbols to proportion	+	—	—	—	—	—	—	+	+	—	—	—	—	—	—	—	—	-	-	-		
38.	Stating hypotheses	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	+	+	-	-	-		
39.	Testing hypotheses	—	—	—	—	—	—	—	—	+	—	—	—	—	—	—	—	—	—	-	+		
40.	Stating procedures	+	—	—	—	—	—	—	—	+	—	—	—	—	—	—	—	—	-	-	+		
41.	Proposing Tests	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	-	-	-		
42.	Formulating problematic situations (fluency)	—	—	—	—	—	+	+	—	—	—	—	—	—	—	—	—	—	-	-	-		
43.	Ditto (flexibeility)	—	—	—	—	—	+	+	—	—	—	—	—	—	—	—	—	—	-	-	-		
44.	Using insight	+	—	—	—	—	—	—	—	+	—	+	—	—	—	—	+	—	-	-	-		
45.	Failure to grasp the essence of the problem	+	—	—	—	—	+	—	+	—	—	+	—	—	—	—	—	—	-	-	-		

3. In another restricted study on the Role of Hypotheses in Solving Problems in Science, R. M. Misra and N. Vaidya devised five Piaget type problems for investigating thought processes among 50 science students of grade X The five problems were supposed to test combinational grouping, formulating problematic questions, proposing tests for determining the nature of the substance and forming and testing of hypotheses (two problems). They were also asked to give their pre-test reactions by answering a series of six different questions. Whereas the questionnaire approach was also used, the experiments were properly displayed and demonstrated in small groups of 10 pupils each. The mathematical structure of these problems after the rotation of factors is reproduced in the orthogonal form. Here, the following mathematical structure appeared :

First Factor : The highest loading on this factor is .9132. The other loadings immediately following this loading in terms of size are .8372, .7969, .7442 and .5652. These loadings are on social, emotional, health. school and home adjustments respectively. Loadings of other variables are small and can be ignored. Hence this factor is hypothesized as General Adjustment Factor.

Second Factor : The highest loadings on this factor are .8232, .6084, .4504 which relate to two variables : Using four digits at a time, Seeing the problem as a whole and Using three digits at a time. The loadings of other variables on this factor are non-significant and hence can be neglected. This factor is, therefore, interpreted as Ability to See the problem as a whole.

Third factor : The highest loading on this factor is .8954 which is on : 'formulating hypotheses.' Other loadings immediately following it are .4685 and .4665 which are on variables : Interest in solving problem, and confidence in the problem. This factor, is interpreted as 'formulating hypotheses.'

TABLE No 9.

Showing the Loadings of Variables for the Five Factors.*

S. No.	Variable	I	II	III	IV	V
1.	Intelligence	—	—	—	0.7503	—
2.	Home Adjustment	−0.6654	—	—	—	—
3.	Health adjustment	−0.7969	—	—	—	—
4.	Social adjustment	−0.8381	—	—	—	—
5.	Emotional adjustment	−0.8372	—	—	—	—
6.	School adjustment	−0.7442	—	—	—	—
7.	General adjustment	−0.9132	—	—	—	—
8.	Newness of the problem	—	—	—	—	0.8300
9.	Understanding of the problem	−0.5937	—	—	0.4079	−0.4473
10.	Felt difficulty of the problem	0.5380	—	—	—	−0.4139
11.	Confidence in the problem	−0.4454	—	0.4665	0.3695	−0.4509
12.	Interest in the problem	—	—	0.4685	0.4925	—
13.	Using two digits at a time	−0.5241	0.4505	—	—	—
14.	Using three digits at a time	—	0.6084	—	—	—
15.	Using four digits at a time	—	0.8232	—	—	—
16.	Formulating hypotheses	—	—	0.8954	—	—
17.	Setting up control experiments	—	−0.4794	—	—	—
18.	Framing questions	—	—	—	−0.6866	—
19.	Proposing tests	—	—	0.3515	—	−0.5942

*Factor loadings below 0.35 are not shown.

Fourth Factor : Here, the factor loadings become very very intriguing. The following loadings .7503 and —.6866 are highly related negative which relate to the variables of intelligence and framing questions of that type whose answers pupils do not know. The highest positive lodings are .4925, .4079, .3695 and .3407, which relate to the following variables : Interest in solving problems, understanding of the problems, confidence in the problems, and using two digits. The loadings of felt difficulty of the problem on immediate presentation is similar in sign as those of the loadings on intelligence and framing questions. This factor is interpreted as Interest in generating difficult problems.

Fifth Factor : Here the highest loading is .8300, which is on variable : Newness of the problem. Scoring on this variable was reversible. So the negative loadings on understanding of the problems and confidence in the problem are explicable. So is the factor loading on the felt difficulty of the problem. In whatever way the various loadings are looked, loadings on the Newness of the problem is very distinct and high.

4. As a part of the ERIC Project study nearing completion, T. S. Sandhu under the supervision of N. Vaidya also investigated the structure of formal thought with the help of Piaget type tasks (loaded with scientific contents) on 986 adolescent pupils (505 boys and 481 girls) drawn randomly among the students of twelve high schools of rural areas taking almost equal number of boys and girls belonging to the age groups of 11 + 12 +, 13 +, 14 + and 15 + studying in grades 6 to 10 respectively.

The following tools were used :

1. Test of Piaget Type Tasks (10 tasks developed by the investigation.)
2. Culture Fair Intelligence Test—Scale 2 (Cattell)

3. General Mental Ability Test (Jalota and Singh)
4. Reasoning Ability Test (Dubey)
5. Space Relations (DAT), (Bennett, Seashore and Wesman.)
6. Adjustment Inventory (Asthana)
7. High School Personality Questionnaire
8. Academic Achievement (From School Records).

The test-retest reliability co-efficients of the ten tasks were ranging from .19 to .53 when taken separately. The co-efficients of reliability of the tasks in combination was found to be .87 which exhibited the dependability of the measure and justified its use in the present investigation.

The co-efficients of validity of the ten tasks against the measures of intelligence, both verbal and non-verbal, and Reasoning ability were found to be ranging from .20 to .56, .23 to .52 and .13 to .45 respectively. The co-efficients of validity when taken for the combination of all the ten tasks were found to be .69, .62 and .56 against the measures of Intelligence, verbal, non-verbal and Reasoning Ability respectively. Thus it is evidenced that the test was sufficiently valid. Factorial validity of Piaget Type Tasks was also computed using factor analysis technique. The range of loadings on Factor I was found to be from .337 to .718 for the ten tasks. It may be identified as a General Factor of Adolescent Thought. It showed the high factorial validity of the tests of Piaget Type Tasks. The relevant data of his study are reproduced below :

TABLE No. 10

Showing the Varimax Rotated Factor Matrix*

S. No.	Variables	FACTORS									
		I	II	III	IV	V	VI	VII	VIII	h^2	$1-h^2$
1.	Intelligence (verbal)	758	—	—	—	—	—	—	—	707	—
2.	Intelligence (Non-verbal)	635	—	—	—	—	—	—	—	532	468
3.	Adjustment	—	—	357	—	—	—	—	—	—	658
4.	(A) Reserved, detached/ warmhearted, outgoing	—	—	514	—	—	—	—	—	—	704
5.	(B) Less intelligent, concrete thinking/more intelligent, abstract thinking	—	—	—	—	484	—	—	—	400	600
6.	(C) Affected by feelings, emotionally less stable/ emotionally stable, mature	—	—	622	—	—	—	—	—	490	510
7.	(D) Undemonstrative, Stodgy/ excitable, demanding	—	—	—	—	623	—	—	—	534	466
8.	(E) Obedient, mild/assertive, aggressive.	—	—	—	625	—	—	—	—	598	402
9.	(F) Sober, serious/enthusiatic, heedless.	—	—	—	—	469	—	—	—	418	582

*Factor loadings below .35 are not shown and all decimal points have been omitted in factor loadings

S N.o.	Variables	FACTORS									
		I	II	III	IV	V	VI	VII	VIII	h^2	$1-h^2$
10.	(G) Disregards rules, expedient/moralistic, persistent	—	—	523	—	—	—	—	—	376	624
11.	(H) Shy, timid/adventurous, socially bold.	—	—	649	—	—	—	—	—	470	530
12.	(I) Tough minded/tender minded	—	—	—	—	—	780	—	—	648	352
13.	(J) Zestful, likes group action/ circumspect individualism	—	—	—	473	—	—	—	—	—	742
14.	(Q_1) Self assured, secure/ apprehensive, insecure	—	—	—	621	—	—	—	—	488	512
15.	(Q_2) Socially group dependent/ self-sufficient, prefers own decision.	—	—	—	—	—	—	—	680	555	445
16.	(Q_3) Uncontrolled, follows own urges/controlled, self-disciplined.	—	—	418	—	—	—	—	—	418	582
17.	(Q_4) Relaxed, unfrustrated/ tense, frustrated	—	—	—	462	—	424	—	—	521	479
18.	Space relations (DAT)	—	—	—	—	—	—	—	-519	433	567
19.	Reasoning ability	684	—	—	—	—	—	—	—	560	440
20.	Classification	573	—	—	—	—	—	412	—	468	532
21.	Grouping of thought	549	—	—	—	—	—	—	—	482	518

S No.	Variables	FACTORS									
		I	II	III	IV	V	VI	VII	VIII	h^2	$1-h^2$
22.	Generalization to arithmetical and algebraic symbols	699	—	—	—	—	—	—	—	559	441
23.	Permutations and combinations	697	—	—	—	—	—	—	—	536	464
24.	Ratio and proportion	517	—	—	—	—	—	—	—	—	690
25.	Formulating probing questions	632	—	—	—	—	—	—	—	—	539
26.	Interpretation and coordination of information	677	—	—	—	—	—	—	—	537	463
27.	Stating & testing of hypotheses	682	—	—	—	—	—	—	—	528	472
28.	Space visualization	493	—		—	—	—	—	—	473	527
29.	Grasping the essence of the problem	—	—	—	—	—	—	593	—	435	565
30.	Academic achievement in Maths	—	666	—	—	—	—	—	—	480	520
31.	Academic achievement in Science	—	718	—	—	—	—	—	—	547	453
32.	Academic achievement in English	—	774	—	—	—	—	—	—	618	382
33.	Academic achievement in Punjabi	—	774	—	—	—	—	—	—	609	391
34.	Academic achievement in Hindi	—	747	—	—	—	—	—	—	582	418

TABLE No. 11

Showing Latent Roots of Correlation Matrix, Percentage of Trace Total Accounted for the Eight Factors

S. No.	Factors	Latent roots	Percentage of trace tota variation accounted	
			Each cemponent	Accumulate p percentage
1.	General intellectual factor	6.5503	19.3	19.3
2.	Academic achievement factor	2.4922	7.3	26.6
3.	Adjustment factor	1.7859	5.3	31.9
4.	Behavioural factor	1.5040	4.4	36.3
5.	Emotional factor	1.1640	3.4	39.7
6.	*Temperamental factor	1.1162	3.3	43.0
7.	*Group Factor of adolescent thought	1.0600	3.1	46.1
8.	*Social factor	1.0037	3.0	49.1

*Rejected by using Scree test as suggested by R.B. Cattell

In this ERIC study, the following eight factors appeared :

1. General intellectual factor;
2. Academic achievement factor;
3. Adjustment factor;
4. Behavioural factor;
5. Emotional factor;
6. Temperamental factor;
7. Group factor of adolescent thought; and lastly,
8. Social factor.

5. In another group of studies, John W. Renner and Anton E. Lawson attempted to investigate the psychological parameters of concrete and formal thought underlying a group of Piagetian problems. The different analyses were done on the different samples drawn from school (7-12 grade) college freshmen and high school biology, chemistry and biology students. Their factor analytic data are reproduced below.

TABLE No. 12

Showing Principal Components Analysis After Varimax Rotation Seventh to Twelfth Grade Students in Oklahoma Public Schools N = 514*

S. No.	Tasks	First factor	Second factor	Precent communality.
1.	Conservation of Solid Amount	- 0.0098	0.7758	60.2019
2.	Conservation of Weight	0.0952	0.7895	63.2426
3.	Conservation of Volume Using Clay	0.4544	0.4402	40.0309
4.	Conservation of Volume Using Cylinders	0.7572	0.0883	58.1118
5.	Elimination of Contradictions	0.7479	0.0466	56.1547
6.	Exclusion	0.7209	0.0413	53.4488
7.	Variance	31.25 p.c	23.84 p.c	

*74 set of incomplete task score were excluded from the analysis.

TABLE No. 13

Showing Principal Components Analysis after Variamax Rotation College, Freshman, N = 143

S. No.	Task	First factor	Percent communality.
1.	Conservation of volume using clay	0.2547	6.4895
2.	Reciprocal Implication	0.7681	59.0014
3.	Elimination of contradictions	0.8053	64.8517
4.	Separation of variables	0.8068	65.0972
5.	Exclusion	0.6818	46.4805
6.	Variance	48.3 p.c	

TABLE No. 14

Showing Principal Components Analysis after Varimax Rotation High School Biology, Chemistry and Physics Students N = 134

S. No.	Task	First factor	Percent communality
1.	Conservation of weight	0.74	55.49
2.	Conservation of volume	0.84	70.40
3.	Separation of variables	0.85	72.13
4.	Equilibrium in the balance	0.80	63.53
5.	Ratio	0.80	63.71
6.	Islands puzzle	0.69	47.97
7.	Variance	62.2 p.c	

What does the Principal Componen: Analysis, one ofthe several methods available for factorization show regarding the mathematical structure of the Piagetian tasks used in their study ? It shows that in general population of the students in grades7-12, the tasks measure formal and concrete thought. In the remaining two samples, the extraction of one factor also shows the existence of formal thought only.

6. Using Principal Component Analysis, Anton E. Lawson considered the mathem atical structure of formal reasoning ability as measured by manipulative as well as written tasks. His data are presented below.

ABLE No. 15

Showing Principal Components Analysis Jsing Varimax Rotation.

S. No.	Measure	Males & Females Factor	Factor Loadings		
			Males	Females	
			Factor I	Factor 1	Factor II
1.	Manipul..tive Tasks Bending Rods	.62	.68	—	.86
2.	Balance Beam	.74	.78	—	.77
3.	Written Tasks Proportions	.74	.74	.92	—
4.	Propositional Logic	.69	.66	.80	—
5.	Combi-natorials	.69	.67	.59	.49

*Factor loadings below .35 not shown

Here, the interesting finding is that the format of the task differentially affects males and females. The distinc-

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tion in mathematical structure between the manipulative and written tasks is lost when data are pooled for both males and females. There is also, however, a side observation of marginal importance, that is, the conservation of volume using clay appears on both factors implying thereby that the problem involves both formal and concrete thought.

7. S. Abramowtiz in his study, on the variable name, ability, noticed a similar case, when he attempted to investigate proportionality skills necessary for its understanding by adolescent pupils as shown by his data given below.

TABLE No. 16

Showing Factor Analysis Results for the Correlative Tests, Average Proportionality Score Size Contrast, Ratio Contrast, and Ability*

S. N.	Variable	Commun-ality	FACTORS		
			1	2	3
1.	Reducing fractions	.48	.68	—	—
2.	Multiplying fractions	.74	.44	—	—
3.	Facility with fractions	.60	.71	—	—
4.	Inverse relations	—	—	—	.48
5.	More than/ Times as much	.36	.41	.36	—
6.	Average proportionality score	.84	—	.90	—
7.	Ability	—	.37	.42	—
8.	Ratio contrast	.41	—	—	.60
9.	Size contrast	.60	—	—	.71

*Factor loading below .35 are not shown.

What exactly is the nature of proportional reasoning ? Is it merely an indicator of abstract thought or a part of general ability ? The results indicate that, although ability insignificantly affects subjects' performances, the effect of task characteristics occurred independent of ability for the type characteristic. Further research is necessitated to separate out the effect of general ability from the acquisition of developmental aspects.

8. In another similiar (doctoral) study using different variables relating to the acquisition of algebraic concepts during the secondary school years is also of interest. The factor analytic data in brief extracted from Joshi's work are reproduced below :

TABLE No. 17

Showing Factorial Structure of Algebraic Concepts (N = 1863)

S. N.	Variables	Factor I	Factor II
1.	Generalised Numbers	.70	—
2.	Directed Numbers	—	.63
3.	Algebraic Equations	.73	—
4.	Brackets	.59	.53
5.	Substitution	.47	.62
6.	Exponents	.59	.37
7.	Graphs	.66	—
8.	Variance	34.8 p.c	14.84 p.c

Using Cluster Analysis Technique, only one common factor appeared on the various concepts included in the study. The Centroid Method, however, indicated the existence of two factors, namely, 'Algebraic Aptitude' and 'Symbolic Substitution'. In the context of this study, it is

now no longer a matter of guessing whether the nature of these two factors covered could be again looked into, against the backdrop of schemes of thought, particularly speaking, using Algebraic symbols (summation, proportion, generalization, application etc.,) The so-called concepts of aptitudes as well as the concepts of problem solving ability do not take us far when internal structures of problems are being analysed into the schemes of thought empirically.

9. There is one serious handicap which really inhibits Piagetian workers. It is the use of 'Methode Clinque' in collecting data, a time consuming approach. So following the ten steps of Burney, Lawson, Longlot, Raven, Shayer and Wharry and Tisher, John R. Staver and Dorothy L. Gabel succeeded in developing a resonably reliable and valid tool called 'Piaget Logical Operations Test' (PLOT) According to the authors, it is an objective multiple choice test with four alternatives for questions and four individual scales.

(a) Conservation of volume by liquid displacement
(b) Separation and Control of variables
(c) Combinatorial analysis
(d) Proportional thought.

Questions remaining more or less the same between the two approaches : individual and group, the principal difference lay only in the format used. Use of video tape was made for presenting one of the cognitive tasks for appraising each of the traits usually indicative of Piagetian formal thought.

The factor analytic data of this study using SPSS Factor employing Hotelling method indicated five factors out of which only three factors were considered significant for interpretation by the authors.

TABLE No. 18

Showing Rotated Five-Factor Pattern of Plot. Clinical Interview and Mental Test Scores

S. N.	Variables	Factors					Commn-ality	N
		1	2	3	4	5		
1	Non verbal IQ	.84	—	—	—	-	.77	68
2	Verbal IQ	.78	—	—	—	—	.74	68
3	Total IQ	.92	—	—	—	—	.93	68
4	Cat-verbal	.87	—	—	—	—	.84	70
5	Cat-quantitative	.71	—	—	—	.43	.75	70
6	Cat-non-verbal	.80	—	—	—	—	.79	70
7	Cat-total	.91	—	—	—	—	.97	70
8	Categorical decision 1	—	.83	—	—	—	.76	70
9	Categorical decision 2	.39	.58	—	—	—	.63	68
10	Categorical decision 3	—	—	—	.85	—	.78	66
11	Categorical decision 4-5	—	—	—	—	.84	.83	70
12	Total categorical decision	.37	.60	—	.49	.46	.99	66
13	Task 1	—	.84	—	—	—	.73	66
14	Task 2	.35	.67	—	—	—	.71	72
15	Task 3	-	—	—	.81	—	.69	70
16	Task 4-5	.35	—	—	—	.82	.91	68
17	Total clinical interview	.38	52	—	.51	.39	.99	66
18	Plot-part 1	—	—	.70	—	—	.51	66
19	Plot-part 2	.53	—	.53	.38	—	.74	66
20	Plot-part 3	.39	—	.61	—	—	.58	66
21	Plot-part 4	—	—	.66	—	.38	.69	66
22	Plot total score	—	—	.92	—	—	1.00	66
	Eigen value	10.67	2.31	1.79	1.52	1.08		
	Percent of variance accounted for	61.4	13.3	10.3	8.7	6.2		

The three factors extracted by them were :

1. General mental ability
2. *The Piagetian Cognitive development
3. Plot (Measurement method as used)

10. The work of M. Shayer over the years has been quite extensive so far as large scale testing in Piagetian context is concerned. Like the work of Robert Karplus and his co-workers, this work also needs to be replicated. In his very recent paper, he poses certain complex problems and suggests certain necessary modalities for tackling some of them. In their replies to one another, both M. Shayer and C. Desforges and G. Brown attempt to clarify one another as if one is trying hard to find a shorter route of touching one's own ear by a longer or shorter route as the case may turn out to be without the knowledge of the mysterious starting reference point vaguely put forward by the Geneva school over the years. Coming straight to the basic issue, Piaget regards operational thinking as a unitary construct having three tiers resting actively over each other, namely, description of behaviour, ascending levels of behaviours determined developmently and the use of symbolic logic to describe these behaviours as if to impose structuralist meta theory. As a first step, it is necessary to determine the correlations among the various Piagetian tasks and later on to determine their mathematical structures, that is, whether all of them measure more or less the same parameters. If the factor loading below $\pm .35$ are ignored as was done in the presentation of other similiar studies, the same is said to have been achieved by the tasks chosen by Shayer.

* assessed through clinical method

TABLE No. 19

Showing Analysis of Five Tasks

S. No.	Task	Factor Loading	Communality
1	Pendulum	0.72	0.52
2	Equilibrium in the balance	0.75	0.56
3	Inclined plane	0.78	0.61
4	Chemical combinations	0.73	0.53
5	Flexible rods	0.86	0.74

TABLE No. 20

Showing Factor Analysis of 11 Written-Task Clusters (Unrotated)

S. No.	Tasks	Variable	Factors 1	Factors 2	Communality
1.	Pendulum	Control of variables	0.64	−0.24	0.47
		Deduction of effects	0.63	−0.22	0.45
2.	Equilibrium in the balance	Proportionality	0.70	0.22	0.54
		Equilibrium of system	0.63	0.28	0.47
3.	Inclined Plane	Proportionality	0.65	0.12	0.44
		Equilibrium of system	0.62	0.16	0.42
		Work interpretation	0 49	0.29	0.32
4.	Chemical combination	Combinations	0.21	−0.11	0.11
		Strategy & deduction	0.67	−0.14	0.47
5.	Flexible rods	Control of variables	0.74	−0.21	0.59
		Compensation	0.68	−0.11	0.48

Even if the Piagetian structures show different parameters, these may as well be equally worthy of intensive & extensive investigations. The need will, however, remain that of producing taxonomy of behaviour for specific areas of the curriculum for developing an integrated theory of curriculum for development. C. Desforges and G. Brown however believe that pursuit of accuracy in identifying Piagetian stages may be counterproductive to the important task of providing appropriate curriculum sequences. The latter work is quite immense, and precisely for this reason, the word 'appropriate' should be the function of what Shayer is trying hard to achieve. Both are trying hard to achieve the same objective by touching one's own ears in the same way as mentioned above in the absence of the reference point for start, which on reflection, Piaget does show. One may boil potatoes in the sun with or without a solar gadget. What is at stake is the value or knowledge of the specific heat of potatoes. The classical psychologists did not succeed in developing a reliable and valid body of psychological knowledge out of which could stem methodology of instruction and education truly linked to mental development.

What ought to be stressed by M. Shayer is that human mind has reality processing mechanism acting on the environment to find the reality. It is possible to identify and place a given individual, broadly speaking, on any one of the tiers of mental development as seen by Piaget. Otherwise, the curricular path is circular. It is agreed that those in the business of education should avoid 'mismatches between pupil and course'. Here M. Shayer need not any longer be apologetic and hence avoid an unneeded verdict :

In the world of chemistry, it was about 1890 that the Chemical Society made a ruling that in future no papers would be published in their journal in which an argument was not established by fresh data. Perhaps it is too early to apply this principle throughout educational research. But perhaps a moratorium might be called on articles of comment and opinion in the area of Piagetian studies. However, wrong headed one may believe a theoretical approach to be, it is only in the design and operation of a crucial test that one gains insight into the nature of the problem, and from such tests something new will usually be reported. It also alters the respect one bears to one's opponents. This area has seen too much heat and too little fresh light.

Even when the so supposed psychometric (or mathematical) light is there, one may wrongly see the consequent true concept based on consensus or otherwise even for about a century or so if one cares to consult the history of science for reaching the true concept is not merely a matter of simply applying scientific methods.

A Brief Resume

What do the above studies show ? They show that the formal stage which appears during adolescence is just being scratched and there is yet to be available a single study which investigates Piagetian thought in full, without violating assumptions very necessary for giving definite point of view mathematically. If some pointers in this direction are considered, the field today appears to present the following picture :—

1. Piagetian tasks as given in the Growth of Logical Thinking as well as tasks inhering a continuous chain of

* See the puzzling ways of science in the appendix.

reasoning, measure what is called 'formal thought'. (Bart, Gabel, Gurein, Lawson, Lovell, Lunzer, Misra, Peel, Renner, Sandhu, Shayer, Staver, and Vaidya).

2. Formal thought is necessary to proportional reasoning (Lovell and Lunzer).

3. The structure of formal thought is bi-factorial, viz. Verbal and Non-verbal (Bart W. M.)

4. Analytic thinking, instinctive thinking and strategy appear during adolescence (Abou Hatab, Austin, Goodnow & Guilford).

5. Adolescent thought shows a form of grouping: Concrete operational and co-ordinating concrete-logical (Gurein.)

6 The factorial structure sex-wise during adolescence is not the same on Piagetian Type tasks presented individually as well as in a group (Lawson and Vaidya).

The picture regarding other factors irrespective of their order of extractions is as follows :

A. Immediate Test Reactions to the Problems on Presentation.

(a) Felt difficulty of the problem
(b) Confidence in the problem } (Vaidya & Misra)
(c) Interest in the problem

B. Piagetian Schemes having loadings on Intelligence.
General Intellectual factor—
(Abou Hatab, Beard R. M, DeLemos, Gabel, MacArthur, Peel, Sandhu, Staver, Tuddenham.

C. Piagetian schemes having loadings on first factor :
Schematic learning general (Vaidya)

D. Academic factor (Vaidya & Sandhu)

E. Practical factor (Vaidya)

F. Other schemes of Thought

(a) Exclusion of variables :
Stating and testing hypotheses (Vaidya)

(b) Seeing the problem as a whole (Misra & Vaidya)

(c) Tackling algebraic symbols
Summation and Proportion (Vaidya)

(d) Using constant difference (Vaidya)

G. Attitudinal Factors

(a) Problem orientation Vaidya

(b) Sensing Problems Vaidya

H. Personality factors

(a) Adjustment (Misra, Sandhu, Vaidya)

(b) Interest in solving

and generating problems (Misra and Vaidya)

(c) Behavioural factor (Sandhu)

(d) Emotional factor (Sandhu)

If the order of extracting factors is strictly adhered to, the slowly emerging picture over the last two decades is presented in the tabular form below.

TABLE No. 21

Showing the Current Status of Factorial Structure of Adolescent Thought

S. No.	Factors	Psychological Interpretation	Authors
1.	First Factor	a. General Intellectual Factor	Abu Hatab, Beard, De-Lemos, MacArthur, Peel, Sandhu, Staver and Gabel, Tuddenham, Vaidya, Vernon.
		b. Schematic Learning General	Bart, Renner, Lawson and Vaidya
		c. General Adjustment	Vaidya and Misra
		d. Formal Operational Thought	Abramowitz, Butterworth, Lovell, Lunzer, Pumphrey.
		e. Exclusion of Variables	Shayer
		f. Attainment Factor	Vaidya
2.	Second Factor	a. Piagetian Cognitive Development	Staver and Gabel
		b. Seeing the Problem as a whole	Vaidya and Misra
		c. Academic Achievement Factor	Sandhu
		d. Adjustment	Vaidya
		e. Practical Factor	Vaidya
3.	Third Factor	a. Piagetian Logical Operations Test	Staver and Gabel
		b. Formulating Hypotheses	Vaidya and Misra
		c. Adjustment Factor	Sandhu
		d. Problem Orientation	Vaidya
		e. Interest Factor	Vaidya

S. No.	Factors	Psychological Interpretation	Authors
4.	Fourth Factor	a. Interest in generating difficult problems	Vaidya and Misra
		b. Behavioural Factor	Sandhu
		c. Sensing Problems	Vaidya
		d. Tackling Algebraic Symbols	Vaidya
		e. Adjustment Factor	Vaidya
5.	Fifth Factor	a. Newness of the Problem	Vaidya and Misra
		b. Emotional Factor	Sandhu
		c. Problem Orientation	Vaidya
		d. Symbolization	Vaidya
6.	Sixth Factor	a. Temperamental Factor	Sandhu
		b. Testing Hypotheses	Vaidya
		c. Using Constant Difference	Vaidya
7.	Seventh Factor	a. Group Factor of Adolescent Thought	Sandhu
		b. Aspect Character	Vaidya
		c. Using Constant Difference	Vaidya
8.	Eighth Factor	a. Social Factor	Sandhu
		b. Aspect Character	Vaidya
		c. Exclusion of Variables	Vaidya
9.	Ninth Factor	a. Combinatorial Grouping	Vaidya
		b. Seeing the Problem as a whole	Vaidya
10.	Tenth Factor	a. Intelligence	Vaidya
		b. Verbal Description of procedures	Vaidya

It should not be forgotten now that factor analysis is a highly mathematical technique. The various factors failing to appear in a definite way should be further subjected to verification by carrying out highly imaginative studies using factorially known tests as reference points. The various tests used should cover as many diverse populations meeting the well intended objective criteria of reliability and validity. Similiary, the sizes of the samples should not invariably be less than four times the number of tests used. After having done this, the growth of factors should be clinically explored and the same be checked to test whether the findings come from different chips of different blocks or the different chips of the same block. Whatever be the nature of these findings, they are bound to alter the already proposed structures of intellect. A periodic table of intelligence like the one in chemistry, if it exists, may become available for the benefit of learning psychologists. Until then, as it is apparent, the studies documented above simply as well as superficially reflect the possible structure of thought as proposed imaginatively by the Geneva school using symbolic logic, a branch of mathematics which strengthens findings in any branch of knowledge in direct proportion to its magnitude of use. Lastly, it will not be out of place to mention that the role of hints and cues in the development of concepts as well as problem solving is yet to be investigated with focus on accelaretion of thought along with the varied modes of presenting subject matters, a very worthy aim of any first class educational system anywhere in the world.

Note

In another study nearing completion, Shri S. C. Jain, at the Regional College of Education Ajmer, investigated Problem Solving Behaviour in Physics among certain groups

of Adolescent pupils of class XI in science (N = 180). He collected data on the following 35 variables: Intelligence, Conservation of volume using metal cylinder, Equilibrium in the Balance, Separation and control of variables, Pendulum task, Total Piagetian Task Scores, Proportional Reasoning (BIS),* Proportional Reasoning (AIS)* Additive Reasoning (BIS), Additive Reasoning (AIS), Creative Thinking (BIS), Creative Thinking (AIS), Conditional Propositional Reasoning (BIS), Conditional Propositional Reasoning (AIS), Formulation and Testing of Hypothesis (BIS), Formulation and Testing of Hypothesis (AIS), Conservation Reasoning (BIS), Conservation Reasoning (AIS), Interactional Reasoning (BIS), Interactional Reasoning (AIS), Conservation and Functional Reasoning (BIS), Conservation and Functional Reasoning (AIS), Separation and Control of Variable (BIS), Separation and Control of Variable (AIS), Total of all the 10 Problems (BIS), Total of all the 10 Problems (AIS), Fluency, Flexibility, Originality, Creativity, Achievement in Physics, Achievement in Chemistry, Achievement in Science.

The Factor Analysis indicated the existence of 12 Factors whose latent roots were greater than unity. Using the Scree Test, the following 3 Factors appeared, namely: Schematic Learning General, Creativity and Achievement in Science.

* BIS = Before Interview Scores.
* AIS = After Interview Scores.

CHAPTER 3

The Research Frontier

A Critical Review

What do the above studies indicate ? They show that even today our knowlege about the nature or structure of formal thought is inadequate. Equally true it is of its development by the classroom teachers. Piaget has firmly hinted that any child is biologically equipped for 'knowing' to a varying degree with the functional invariants of organization and adaptation. Studies are, therefore, needed urgently which blend both the psychometric and developmental aspects of thought under varying conditions of environment. Excluded also are not the personality factors. Other factors like the study habits, pupil-teacher interaction and mode of father-son dialogue can also be included. Pitting external structure of problems against schemes of thought hypothesized as well as drawn from the problems actually administered which, in fact, reflect their internal structure along with the other outside variable, hinted above will go a long way in clarifying the complex nature of formal thought. Investigation of specific concepts for specific purposes is also equally revealing but it may not throw adequate light across the varied content matter. A case in point is the detailed investigation of the concep of potential by Sixth Form students which included some of the significant aspects like electrostatic, gravitational, work done and energy. Both practical and theoretical situations (N = 25) were designed along with a highly detailed scoring key. Result: First, advanced students had not firmly formed this concept. Secondly, Principal Component Analysis showed strong intellectual/educational component while varimax rotation isolated two factors associated with 'potential' and 'work done' respectively. Similiar studies by other authors need to be extended further

for romoving cowdung in the field. A factorial study of formal thought, training of thought included, should include the following outside variables as a first step.

1. Schemes of thought : Summation, proportion, combinatorial grouping, exclusion of variables, correlation, chance, eliminating contradictions, conservation (volume and weight), and the INRC structure.
2. Verbal and non-verbal tests of intelligence, tests of abilities and personality tests.
3. Creativity variables of the variables drawn from the structure of Intellect as propounded by J. P. Guilford.
4. Influence of training, and the other environmental factors.

Reality may still evade us for Piagetian Psychology is 'closely linked to a certain form of epistomology'. To quote Barbel Inhelder

> Once understood, this form of psychology appears to be that which best suits genetic psychology, as both are essentially constructivist. Constructivist implies that knowlege is not acquired merely under the impact of empirical experience as suggested by behaviourists' theory, although, of course, such impact is not entirely excluded from the process. It is also opposed to innate theory, to which it seems recourse is frequently had today (maturation being a factor which intervenes but not exclusively). Constructivism emphasizes the child's or the subject's activity during the course of development, in other words, everything derives from actions and is evidently translated into coherent and logical thought operations.

The factor analysts are, therefore, now needed to separate the chaff from the grain by developing imaginative techniques especially when the increasingly high speed computers are at their door step.

Let us now make a few critical observations as well. The researches of several workers in the field indicate firmly that our konwledge about the nature and development of formal thought for day to day classroom teaching is incomplete in circumscription, description, organization, knowledge, application and evaluation (Elkind, D; Inhelder, B; Jain, S. C; Joshi, J. N; Karplus, E; Karplus, R; Lawson, A. E; Lovell, K; Lunger, E. A; Modgil, C; Modgil, S; Padmini, M. S; Peel, E. A; Piaget, J; Renner, J. W; Ross, R. J; Sandeu, T. S; Wollman, W and Vaidya, N.) It is not simply sufficient to know that of the fifteen problems used by Piaget and Inhelder in 'The Growth of Logical Thinking' measure the same parameter, that is, formal thought or sometimes they may appear in concrete thought as well. Equally so, whether a given scheme of thought (or several other ones) is an integral or detachable component of either or both aspects of intelligence : Verbal and Non-Verbal. Thirdly, whether the various schemes of thought appear more or less simultaneously during adolescence. And fourthly, how do they manifest themselves during the proceeding stages of mental development ? Fifthly, whether sex differences appear when Piaget type tasks are translated into paper pencil tests. The basic problem is to identify and test the growth of various schemes of thought right from their moments of origin to their merger in relation to personality variables and other clear cut enviornmental variables. Technically speaking, Jean Piaget has not only utilized but suggested a scheme of research in which are clearly indicated a given scheme of thought along with the corresponding set of experiments. As experience grows, other schemes of thought along with their corresponding experiments drawn from other areas of knowledge can also be added. This scheme of research is now reproduced below :

TABLE No. 22

Showing the Schema and Their Corresponding Experiments

Schema	Experiment
Combinations	Chemical Combinations, in a system containing a substance to be coloured, a dye, an inhibitor, and a neutral agent.
Proportionality	Equilibrium on a balance beam, where the multiplicative relation between length and weight must be dealt with.
Correlations and probability	Dicovering the relations between a pair of imperfectly correlated variables (hair and eye colour)
Conservation beyond empirical experience	Conservation of movement in a system containing some friction, i. e., rolling balls on a horizontal plane.
Inversion and reciprocity co-ordinated in maintenance of equilibrium.	Behaviour of liquid in communicating vessels (equality of water levels, relation between water pushed out of one tube and into the other).

Schema	Experiment
Mechanical equilibrium	Hydraulic press (a more quantitative version of the preceding).
Co-ordination of two reference systems	Snail moving on a moving platform.
Equilibrium of work, mechanical proportions	Behaviour of wagon on variably inclined plane counter balanced by variable weight on pulley.
Geometrical proportionality	Predicting size of shadow cast with objects varying in size and distance, screen and source varying in distance.
Compensation of interacting variables	Behaviour of balls on rotating platform, relation between weight and distance from centre in determining centrifugal motion.

In this context, it is of passing interest to mention the approach actually followed by N. Vaidya while investigating several aspects of adolescent thought with special reference to physical sciences.

TABLE No. 23

Showing Schemes of Thought and the Corresponding Problems and Experiments along with their Co-efficients of Reliability and Validity.

S. No.	Schemes of Thought	Problems and Experiments*	Reliability co-efficient	Validity intelligence	Co-efficients Problem solving ability
1.	Using constant difference	Height problem., Positive Constant Difference Problem., Negative Constant Difference Problem, Rectangle Problem., Spring Blalance Problem.	·89	·83	·60
2.	Using Summation	Hotel Problem., Rectangle Problem., Rectangular Cubes Problem., Fish Problem., Spring Balance Problem.	·91	·88	·37
3.	Using Proportion	Proportion Problem., Hotel Problem., Rectangle Problem., Rectangullar Cubes Problem., Fish Problem., Spring Balance Problem.	·86	·91	·67
4.	Beaker Combinations	Beaker Problem.	·67	·64	·35

S. No.	Schemes of Thought	Problems and Experiments*	Reliability co-efficient	Validity intelligence	Co-efficients Problem solving ability
5.	Using two digits at a time.	(Combinatorial)–Digital Problem.	·96	·07	·24
6.	Using three digits at a time	(Combinatorial)–Digital Problem.	·95	·96	·53
7.	Using four digits at a time	(Combinatorial)–Digital Problem.	·98	·91	·51
8.	Generalization to algebraic symbols (Summation)	Positive Constant Dfference Problem, Negative Constant Difference Problem.	·88	·59	·50
9.	Generalization to algebraic symbols (Proportion)	Proportion Problem.	1·00	·45	·22
10.	Stating Hypotheses	Flow of Water Through a Tube Problem.	·68	·86	·69
11.	Testing Hypotheses	Flow of Water Through a Tube Problem.	1·00	·35	·08

S. No.	Schemes of Thought	Problems and Experiments*	Reliability co-efficient	Validity intelligence	Co-efficients Problem solving ability
12.	Stating Procedures	Fish Problem., Spring Balance problem., Rectangular Cubes Problem.	Variables do not change		
13.	Proposing Tests	Proposing Tests Problem.	·68	·73	·32
14.	Formulating Problematic Situations (Fluency)	Formulating Questions Problem.	·91	·84	·40
15.	Ditto (Flexibility)	Formulating Questions Problem.	Varia bles do not change		
16.	Insight	Counting maximally Ractangles Problems. Nine Dot Problem.	·90	·31	·11
**17.	Failure to grasp the essence of the Problem	Questions Inviting Wrong Answers.	·73	·53	·40

* Each problem is composed of several diverse problem solving processes.

** Higher score means grasping the essence of the problem.

It is now not far fetched to see other areas of knowledge coming under investigations in the Piagetian context. Such examples are : biology, physics chemistry, mathematics, earth sciences, astronomy and descriptive subjects like history, geography, home science, economics, etc, if this interest in research is made more varied. Specifically speaking. R. J. Ross influenced by the work of Jean Piaget and his co-workers has tried hard to state the Piagetian problematic territory as clearly as possible. To quote him :

> The empirical studies show that formal operation is far from universal. Higgins-Trenk and Gaite (1971) observed from a conservation of volume task that 43 per cent of high school students operated formally Jackson (1965) found 44 per cent of 13-year-olds, 50 percent of fifteen-year-olds responding at the formal level and Tomlinson-Keasey (1972) reported percentages of 32, 67 and 54 for 11-year-old girls, for college women and for adult females. When the more stringent III-B criteria were applied, percentages dropped to 4, 23 and 17 respectively. However, Case and Collinson (1962), Elkind (1966), Higgins-Trenk and Gaite (1971), Jackson (1965) and Yudin (1966) reported a significant increase in formal operational thinking during adolescence, especially between 11 and 15. However, Ross elaborates that statistical technique can lead to differing emphases. Employment of variance analysis or a non parametric test allows the reporting of a significant increase with age in the number of subjects operating at the formal level. At the same time, an examination of the absolute percentage of people operating at the formal level may suggest a different interpretation. Ross cites Tisher (1971) who reported a significant increase in the number of adolescents moving from the concrete stage to formal stage but acknowledges

that even in the oldest group less than 50 percent were judged to be at the formal stage.

Further, the nature of the task appears to influence experimental results and it has yet to be established whether Inhelder and Piaget's diverse array of tasks are measuring the same construct. Ross cites Evans' (1971) comment with respect to Lovell's (1971) statistical investigation of formal operations, the W, the coefficient of concordance was 'a fairly low correlation and means that there is a good deal of disparity.' Ross considers that Hughes (1965) and Lovell and Shield's (1967) factor analytic studies need replication with larger sample sizes. Field and Cropley (1969) found a significant correlation between science achievement and formal operational performance and therefore the posibility of formal task being linked to specific subject content and thus differentially affecting a particular social group, limits their generality as measure of intellective ability.

Piaget's (1972) introduction of the concept of aptitude as an explanatory device for the low occurrence of formal operations and reviewing formal operational structures to be more accurately assessed within a particular aptitude or vocational domain, leads to the realization that he is attempting to disassociate the formal operations from a specific and limited subject matter at the same time that he is linking them to the particular aptitude domain of the individual. Ross agrees with Piaget that further experimental verification is needed for the conclusion that the formal structures are probably common to all individuals but are applied differently by each person in his particular activities. Ross however adds the proviso that 'Assuming that the formal structures are manifested within a particular aptitude context, it will first be necessary to isolate the superior aptitude of each individual and then present a

formal task congruent with that aptitude, thereby making research more complex than before.

As already referred to, the research frontier hinted here presents several problems some of which are dependent on each other for serious study by the futuristic research workers. Others may be found out illusory. They should not feel inhibited in the light of past efforts or look upon faults in the problems, as faults found in the rocks where the continuity of one rock shades into another or is lost thereby deceiving the geologists. These are insuperable difficulties. They should, therefore, strike hard with their hammers and consequently penetrate deep with their drills into the problematic phenomena. It is probably about this that Plato spoke when he talked of 'Cutting Problems at the Joints' more than 2000 years ago.

CHAPTER 4

The Educational Implications

Whereas the contributions of the Geneva School are immense, it is more or less, specifically silent on educational implications. According to Howard E. Gruber and J. Jacques Voneche, the importance of self activity in intellectual growth, type of thinking manifested by the child's thought in relation to certain universals like nature of objects, space, time, motion and speed etc. and the presence of certain underlying 'pervasive logico-mathematical structures in these diverse areas' certainly suggest the following four models of teaching learning process for consideration and utilization in varying proportion by the practicing teachers while 'schematizing the use of materials and exercise on the one hand, and the type of social relationships on the other'.

Modles of Teaching

1. Taos : respect for the individual artisan. Child learns through self activity. He chooses according to his own taste certain ac ivities out of the many such activities, suitably designed and sequenced by the teacher. As the child progresses further, these activities and materials or tasks can be replaced by exercises achieving more or less the same ends. In case these activities or exercises are undertaken at school they are surely going to have social cognitive goals like imitation, decentration, moral judgment. Thus the child gradually learns to accept his responsibilities for his own intellectual development.
2. Paris : Pleasures of discussion. Here, physical activities, materials or even tasks are unimportant. They have only limited value especially when an argument or point of view is to be tested or checked. In this model, there is an intelligent dialogue between equal partners or more or less equal partners.

3. Athens : The Socratic method. Here there is an intelligent dialogue between the teacher and student. It is the teacher who dominates by putting questions or posing problems concretely as he thinks fit. Attempt is made to enhance pupil knowledge. Thus the gap between the pupil knowledge is reduced through suitable means selected by the teacher. To stress, the dialogue remains the main tool, with no intention to create a 'relationship of equality'.

4. Eldorado : Here both the teacher and the pupil participate in the processes of concept formation, application and discovery. Rich experiences with immense possibilities of exploration in several educational directions are a must. The child watches her teacher at a distance where the latter shares his knowledge by in fact living it. To put in other words, it means that the child is in a position to imitate her teacher.

Cycles of Learning

Prof. Robert Karplus and his co-workers have recently suggested a highly imaginative and creative sequence of learning experiences, viz. exploration, invention and discovery. In this triple process of presenting concrete tasks or physical activities, the child is first expected to handle the semi-structured experimental materials freely; later on, he is expected to develop the concept unaided and lastly, to apply the concept formed in several diverse problematic situations. It is yet a moot point whether accelerated thinking may or may not result (Piaget is quite silent on it). One can safely say that lot of work is involved when any cognitive structure is widened, deepened and 'linked more coherently' with other structures. It is further possible to extend or modify the affective range in varied contexts like pleasure, pain and the actual context in which it is exercised. Gruber, H. E. & Voneche, J.J. rightly add

that one of the central meanings of ego strength must certainly be that powerful cognitive structures do not become inactive in emotionally laden situations. And finally suggest that there need not be any Piagetian dogma, for teaching means creating learning situations in which children first discover concepts and apply them later on. There is a corresponding piece of advice for the teachers as well, i. e., they must invent and reinvent what they are doing.

Conditions for Formal Learning

It is a philosophical piece of advice. The properly and firmly tested piece of advice is yet to become available because we do not yet know the condition under which learning takes place. One is not sure when learning conditions are specified in the sense whether they lead to guaranteed successes. However if teachers learn to invent and reinvent their methodology of instruction and education, Kimball has suggested certain measures which if undertaken, hopefully go a long way in providing atmosphere for the acquisition of formal operations. There in his words while teaching formal operations are : The individual that is to be introduced to formal operations must be facile with concrete operations; able and experienced in manipulating materials: capable of formulating generalizations; conserves numbe. as well as area, shape, substances and volume; classifies according to multiattributes; identifies variables; identifies relationships and manipulates them in novel ways, exhibits reversibility. An environment that encourages formal thought must now be provided.

1. A facilitator who himself is formally operational. He is a learner as well as a teacher. He allows many divergent responses, keeping those which meet the criteria set up. He accepts each person even though that person's finding may not 'fit' or 'work' in this situation.

2. Surroundings are rich in materials and ideas that encourage manipulation (first hand), interaction and inter-change.
3. The learning atmosphere allows for risk taking and creativity.
4. Constructive intervention takes place between learner and enviornme.t ar d amongst learners in a sensitive manner.
5. Criteria for acceptable results are set up by all learners in the situation.
6. 'Messing around' is encouraged so that learners can 'feel' the problem as well as 'think' it.
7. Learner is actively involved in coming to conclusions and testing conclusions in new situations. His activity is reinforced 'neurally' by the teacher.
8. The type of materials present are chosen because they encourage and suggest formal thought, not stand in the way of it. Learning is tailored for the individual.
9. Evaluation tools are available to measure formal thought so that feed back is immediate and personal as well as shared.
10. The learner is placed in a situation of mild conflict and controversy with the materials and the ideas of his colleagues.
11. The training is from simple to complex : from concrete to abstract.
12. Environment allows for curiosity to be met with open discussion, mutual criticism, or support. Final arbitration is by replication with materials.

Concluding Statement

Lastly, the work of Jean Piaget simply shows us the direction which if followed will make our classrooms more and more responsive to the development of formal oper-

ations where the mode of instruction is definitely going to play a decisive role. It is so because the evidence is accumulating fast that majority of adolescent pupils even approaching adulthood do not exhibit formal thought as propounded by Geneva school. If such steps are taken, the teachers are bound to encounter problems not hitherto met which, if they solve through dialogues amongst themselves, may generate new formats in methodology of instruction. There will be gains on the pupils' side as well because they will be then developing a sort of positive as well as open relationship with their teachers. John Holt, then need not remark that the only difference between the bright children and the dull children is simply this, that is, the dull children forget before the examination and the bright children after it. School then ceases to be a place where children simply learn to mess around and achieve nothing in the process. The Geneva school offers insights into the intellectual functionings of children where the energetic and the enquiring teachers may try their hands in the context of educational models mentioned above. If this line of argument is accepted, the teacher has a role to play where he ought to intervene intelligently in pupils' yearnings, namely, security and adventure. Depending upon the objectives of his professional time to be spent with children, he has to play any one of the five overlapping roles as put forth by Ronald G. Good in his book, 'How children Learn Science' :

1. Lecturer and question-answerer
2. Discussion leader
3. Activity director and evaluator
4. Activity facilitator
5. Co-investigator

The outcomes of instruction could be both verbal, and non-verbal; and the blend of the two.

APPENDICES

The Piagetian Journey (1896-1980)

Introductory

Biographical Note

It is said that a really great man is one who halts progress, at least in his subject for about four centuries. In his life time, Piaget nearly won this distinction because he was no stranger in academic circles all over the world. Whereas his works were both acclaimed and ridiculed by the academicians, he built up his own Piagetian history, caring little either way, single handed, which, later on, became world famous as the Geneva School. About the slow and grudging acceptance of his work, which hardly clicked contextually among workers when it came to intense criticism, he has frequently said:

> I am a naturalist and biologist by training interested in epistemological problems without ever having undertaken formal study (nor passed any examination) in psychology, my most central concern has always been to determine the contributions of the person's activities and the limiting aspects of the object in the process of acquiring knowledge. Fundamentally, it was the wish to resolve this problem using the experimental approach that brought me into the field of developmental psychology. But it follows logically since this point of view is not often held by psychologists in general—and even less by child psychologists that those who read my work find themselves confused (3).

Piaget was born on August 9, 1896 in Neuchatel, a French speaking university town in Switzerland. Right, from his early days he was very studious. He had also a lonely childhood because of the emotional outbursts of his neurotic mother and the detached attitude of his historian Father. Interestingly enough, he wrote his first scientific

paper at the age of ten for which he was offered the post of a curator at the local Natural History Museum, which offer was withdrawn immediately in view of his low age. He obtained his doctoral degree in Zoology at the age of 22 on 'Mollusks of Valias.' By observing, dissecting and experimenting with them, he applied the knowledge so gained to the origin of knowledge as it grew historically and the way it developed inside the head of child scientifically. In this encounter, he did not know what he was to accomplish in his life. So he flirted with several ideas and read deeply and reflectively in philosophy, psychology, sociology and history of science etc. As his intellectual thirst did not quench, he went to Paris for pursuing clinical research at the Alfred Binet Laboratory. While he was paid for standardizing Burt's tests of reasoning, he deeply engrossed himself with pupil questions and errors which in a way provided him with missed insights into intellectual development.

Another scientific paper by him in this area won him in 1921 the job of Director of Studies at the J. J. Rousseau Institute in Geneva, a position which offered him ample opportunities for research and reflection. From this moment onward, he never looked back and was actively associated in different capacities with the following few universities like Neuchatel, Lausanne, Sorbonne, University of Geneva, as well as with the International Bureau of Education and Centre for Genetic Epistemology. He taught and researched in philosophy, psychology, sociology and scientific thought. He also co-edited two journals, namely, Archieves de Psychologie and Revue Suisse de Psychology for several years. He spent a lot of time daily on writing articles, papers, monographs & books. Although very busy, he was still approachable and tolerated fools with courtesy and pleasure. At other times, he could be

equally cold. He earned respect and regard abundantly from those who worked closely with him. In 1964, V. N. Rockcastle observed him quite closely when Jean Piaget guided a workshop on Cognitive Development under the auspices of the National Science Foundation of U. S. A. He found him little sophisticated, despite his keen international status. He loved simplicity, possessed a sense of humour and showed a twinkle in his big eye even when in disagreement. He knew the art of travelling light, that is, putting a small bag into a large one secretly for carrying back home his very dear and lovely molusks and no 'Foren' goods. In 1971, he became Professor Emeritus of the University of Geneva. He retired from teaching but, not at all, from research. He continued to write until the end of his life.

His Chronology of Publications

Undoubtedly, Piaget wrote profusely in his life time in French. How he wrote is a matter of conjectures to the outsiders. Consider his following publications translated in English, written singly and, in collaboration with others, starting from 1923.

1.	The Language of Thought of the Child	1923
2.	Judgement and Reasoning in the Child	1924
3.	The Child's Conception of the Physical World	1926
4.	The Moral Judgement of the Child	1932
5.	The Origin of Intelligence in Children	1936
6.	The Construction of Reality in the Child	1937
7.	The Child's Conception of Numbers	1941
8.	The Child's Construction of Physical Quantities	1942
9.	Play, Dreams and Imitations in Childhood	1945
10.	The Child's Conception of time	1946
11.	The Child's Conception of Movement and Speed	1946
12.	The Child's Conception of Space	1948
13.	The Psychology of Intelligence	1950

14.	Play, Dreams and Imitation in Childhood	1951
15.	The Origin of Intelligence in Children	1952
16.	Logic and Psychology	1953
17.	The Construction of Reality in the Child	1954
18.	The strategy of Genes	1957
19.	The Growth of Logical Thinking from Childhood to Adolescence	1958
20.	The Child's Conception of Geometry	1960
21.	The Early Growth of Logic in the Child	1964
22.	Mental Imagery in the Child	1966
23.	The Mechanisms of Perception	1969
24.	The Psychology of the Child	1969
25.	Science of Education and the Psychology of the Child	1969
26.	Genetic Epistemology	1970
27.	Biology and Knowledge	1971
28.	Insights & Illusions of Philosophy	1971
29.	Psychology and Epistemology Structuralism	1971
30.	Understanding and Causality	1974
31.	The Origin of the Idea of Chance in Children	1975
32.	The Grasp of Consciousness	1976

Surprisingly enough, this list is not complete for he has authored some more books whose dates of publication could not be ascertained: Principles of Genetic Epistemology; Memory and Understanding; and Success and Understanding.

The Cycle of Intellectual Development

From the above mentioned chronology of his publications, it is abundantly clear that it is even difficult to read Piaget at one go. Perforce of circumstance, one has to start at one of the corners, go ahead on his Voyage of Discovery and see first hand the vastness of his difficult work. Having flirted elsewhere and, later on, delved deep in the Piagetian territory, the Indian author of Some

Aspects of Piaget's Work and Science Teaching remarked in a poetic fancy : "I have loved Piaget more than I have loved my wife for his creative work and intriguing system which are as exciting as space travel and research." (14) He is not one Piaget but several if one takes into account the multisided aspects of his work. He is not only very much aware of the theoretical implications of the work and attacking problems from one's weakest points in the eyes of others, but also, he is quite imaginative and capable of choosing complicated techniques and methods relative to his select intellectual problems defined in the widest possible framework surpassing psychometry clearly and even development. Over a period of sixty years or so, he and his co-workers have contributed immensely to the whole field of psychology : perception, reasoning, intelligence, dreams, moral development, space, time, play, thinking from early childhood to late adolescence and other highly varied problem areas, for example, physics, mathematics, biology and logic. From his highly theoretical, abstract and imaginative frame of reference, he has devised highly original physical experiments which have won him world-wide acclaim (e. g. conservation experiments) and are admired highly by the researchers on cognitive development. Result ? His theoretical frame of reference, an incomprehensible entity to many, possibly of less calibre, became a casualty rather than a causality. Here, even Professor Flavell, is no exception, the first best American officer Piaget had in the 60's.

According to him, intelligence developes in four stages. These are : Sensory Motor (0-2 years), Pre-logical (2-7 years), Concrete (7-11 years) and Formal (11-15 years). Recently, he has hinted at the possibility of the fifth stage which covers the period from 15 to 20 years after taking into account factors like aptitude variations and commit-

ment to individual careers against the backdrop of now less prominent general intellectual development. The first stage is of little educational significance but, at the same time, is at the bottom of all the succeeding stages. In the second stage, the thinking or reasoning is transductive, that is, from particular to particular. At the third stage, reality dominates thinking which is reversed at the fourth stage. Here, the adolescent pupil can set up all sorts of hypotheses, test in varied contexts through control experiments, and may eventually pick up the various elements of scientific methods. Piaget then links up these stages with his self designed symbolic logic, a branch of mathematics, for analysing intellectual behaviour. Further, computer science is brought in and the whole scientifically discovered processes of pupil thought are very carefully hooked to the origin of knowledge, and, hence, experimental epistemology. It is then not difficult to see the passage of thought from stage to stage and the attainment of the highest rationality, potentially speaking in man.

Other Related Works

Incomplete conjectural research work very frequently raises more questions than it answers. His work met the same destiny as is seen by the diverse publications in different areas of work which appeared in the sixties and seventies. Whereas the work of Piaget is supported in principle by the various authors, no systematic statement about the testable hypotheses so far appears for judging the weakness of the system. Even the main research frontier of logical thought now extended upto 20 years of age awaits immediate invasion. Now consider the chronology of publications by others which runs parallel to his main work.

1. Oaks N. E. Children's Explanation of Natural Phenomena 1947

2.	Isaac S.	Intellectual Growth in Young Children	1948
3.	(i) Inhelder Barbel	Experimental Reasonings of Adolescents	1955
	(ii) Inhelder Barbel	Learning and the Development of Cognition	1974
	(iii) Inhelder Barbel et al (Edited)	Piaget and His School	1976
4.	Keats J. A.	Formal Concrete Thought Processes	1955
5.	(i) Lunzer E. A.	Recent Studies in Britain Based on the Work of Jean Piaget	1960
	(ii) Lunzer E. A.	On Children's Thinking	1970
6.	(i) Peel E. A.	The Pupils' Thinking	1960
	(ii) Peel E. A.	The Nature of Adolescent Judgement	1971
7.	Lovell K.	The Growth of Basic Mathematical and Sientific Concepts	1961
8.	(i) Flavell J.	The Developmental Psychology of Jean Piaget	1963
	(ii) Flavell J.	Cognitive Development	1977
9.	Ripple R. E. & Rock U. N.	Piaget Rediscovered	1964
10.	(i) Wallace J. C.	Concept Growth and the Education of the Child	1965
	(ii) Wallace J. C.	Stages and Transitions in Concept Development	1972
11.	Almy N.	Young Childrens' Thinking	1968
12.	SCIS	Curriculum Study Source Book	1968
13.	Siegel I. E.	Logical Thinking in Children	1968

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Piaget was a great polymath who did not at all hesitate asking big questions considered definitely out of bounds by the contemporary twentieth century psychology and provided experimental answers to the philosophical questions, contrary to the philosophical tradition of Socrates, Plato, J. S Mill, John Locke, David Hume and Immanuel Kant, to mention only a few. The spate of the above mentioned books within the last two decades shows that the Piagetian territory is nascently fermenting, actively healthy and kicking, and definitely productive, either way, (because failure in science is not a rare event) and hence, can't be wished away for he showed relationships between his experimental epistemology and the entire human sciences.

Everlasting Nature of His Work

It is difficult to answer this question. Having been trained as a physicist, I did not know what I was doing while working on thinking : Concept development, problem solving and their underlying mathematical structures. Through sheer chance, I came across one of his books and found it too annoying. If one is really serious and works on the problem from its first fundamentals, one curiously enough, finds oneself playing on a sea-saw with Piaget. The intellectual swing is experienced and then, one gravitates towards or away from his territory. This is not the way of answering the above mentioned academic question. But here, it has to be so. Why ? Whereas his journey had earlier begun from Mollusks to Moppets, mine began from the failure of understanding the deficit learning situ-

ations through teaching methodology, deaf, to the mentality of children. To illustrate the difficulties :

Imagination, ingenuity and skill open up several diverging perspectives for any age old problem stated in its very fundamentals. From the study of Mollusks, he formulated problems productively for the theory of knowledge for the next six decades by observing them, 'ferreting out their notions of time, space, numbers and ethics as he sprawled on the ground shooting marbles and playing other games' in the words of John Leo (6). Thus he challenged the then current wisdom about child development. As the time went by, he became a legend in his time with eyes ablaze as he tackled a series of intellectual problems, one after another. Consequence ! The splitting of Piaget ! He is Piaget Number One when one considers his slipshod work on cognition. He is Piaget Number Two if one considers his truly scientific work on perception. He is Piaget Number Three when one considers the educational implications arising out of his work. He is Piaget Number Four when he begins to revise his ideas very fast in the light of after thought, criticism, contradictory data and self critical reflection. He is Piaget Number Five when he lifts a particular problem from its lowest plane to its highest epistemological plane where by a sort of parallelism is established the way the idea was historically developed on the one hand and the way it arose scientifically in the child's mind right from its moment of inception. He is Piaget Number Six in the eyes of those who imagine that the basic Piagetian tenets cannot be proved wrong. He is, in the end, Piaget the Miscellaneous when he flourishes on misunderstandings raked up by those who do not understand him, or accuse him of not solving those problems which he did not propose to do. Whatever be the elephantine aspects of Piagetian work, any one who came close to him

affirms to his greatness after all because 'it was a famous victory' at its best or the so called flop at its worst. The answer to the question raised above can now be answered as best as possible as follows :

1. "Nature desires that children should remain children before they are men", echoed Rousseau. Piaget made this clear distinction scientifically, using Method Clinique, saying thereby, that child is not a little adult.
2. He provided the psychological theory of operations which developes in stages. Operations are actions which are internalized and reversible. They can be thus distinguished from simple action or goal directed behaviour.
3. There is constant interaction between the organism and the environment out of which develop intellectual structures sequentially. Thought accords with things, actions, and even with itself in the inseparable functioning of organization and adaptation.
4. Equally proficient in zoology, logic and epistemology, he provided rich vocabulary having very deep meanings : adaptation, centration, conflict eqilibration, isomorphism, INRC group, operation, reversibility and scheme of thought etc.
5. He brought out clearly the limitations of associationism structuralism, functionalism, behaviourism, and Gestaltism. He developed experimental epistemology. Also he did not favour acceleration of thought.
6. In the Growth of Logical Thinking From Early Childhood to Late Adolescence, he propounded an open research frontier which is yet to be verified as well as tested.
7. His theory does not generate sharp hypotheses which can be tested easily by psychometry. The factorial interpretation of his work is just coming up.

Contradictory Comments

It is said that the five blind persons of Hindustan could not reconstruct the whole elephant for their individual observations lacked synthesis. Similarly, understanding Piaget in toto is a difficult enterprise. By putting aside the two variables of feeling and doing, he concentrated on the variable of cognition which he regarded as fundamental. While working with children, partly speculatively and partly experimentally and that also unscientifically, he interpreted the developing House of knowledge of an Epismetic (Universal) child by giving it an epistemological base which was objected to by the epistemologists. Conseqently, he put up his own web by picking up threads from the various sciences in the name of so called synthesis called compound in contrast to mixture in the language of chemistry. It is this compound which is highly sensitive to the past knowledge and is also a competitive construct of the things likely to be. It is, therefore, least surprising that his work has evoked diverse criticism. So by his critics, he has been described as :

> Discoverer or an unsuspected dimension of truth (Isaac Nathan); Mastering at developing the theoretical implications of his work (Hunt); Zoologist by training, an epistemologist by vocation and logician by method (B. Inhelder); in continuation psychologist by accident (D. M. Hyde); educator by mistake (N. Vaidya); an indelible colourizer (J. Flavell); capable of hypothesizing or producing child following any system of logic (K. Lovell); a sort of bull's eye hitter (N. Vaidya) having 'scant regard for statistics, standardized tests and procedures' with speculation thrown in (Eric Lunzer) and Le Patron as lovingly called in Geneva (4).

Even Jan Smedslund could not help commenting after having been ardenalized by the Piagetian territory for about eight years :

> I probably owe more to Piaget than to any other single psychologist, yet I have become increasingly critical of his theoretical system. Working with children during my Piagetian period, I felt constrained to be detached, one sidedly cognitive and entirely focused upon certain very abstract aspects of performance. What mattered to me then were the formal logical structures rather than the concrete living children in their total life situations. I tended to ignore the artificiality of the tasks and I was oblivious of the political implications of my work as supporting a one-sidedly intellectualistic and school-centred ideology. I doubt that practitioner can be a useful psychologist if he remains within that tradition. Piaget has contributed a wealth of brilliant and penetrating insights but they must be incorporated into a view of psychology which can be lived or practised rather than merely written and talked about in academic settings (4).

Borrowing from physics, it is simply a case of polarization after his interaction with Piaget because any logico-mathematical structure could be traced from its moment of origin under emotive conditions as well. It is precisely for this reason that Piaget ran away, literally speaking, from psychoanalysis. Even this remark may not hold true when he wrote another thought provoking book on the Grasp of Consciousness towards the fag end of his life where he made children perform interesting activities, for example, Walking on All Fours and the Flying Balloon with a view to understand how consciousness, preconscious and unconscious hang together when children, varying

over a very wide age range, begin to reflect on their actions and continual reflection on each other leading to conceptualization, understanding and failure. His use of vocabulary has specialized meanings. For illustration, take the word : Cognizance which he used when any subject fully becomes conscious of a situation. To quote Piaget :

> The study of cognizance has led us to place in the general perspective of the circular relationship between subject and object. The subject only learns to know himself when acting on the object, and the latter can become known only as a result of progress of the actions carried out on it. This explains the circle of the sciences, of which the solidarity that unites them is contrary to all linear hierarchy. Furthermore and most importantly, this explains the harmony between thought and reality. Since action springs from the laws of an organization, that is, simultaneously one physical object among many and the source of the acting, then thinking subject (10).

Here is provided the inside developing picture of the black box nearly ignored by the behaviourists within the S-R paradigm trying hard to push back Sigmund Freud, J. B. Watson, B. F. Skinner, Robert Gagne, Max Wertheimer, Karl Duncker, J. S. Bruner and Ausubel. The same remark appears to apply to the advocates of heuristic method, inquiry training, guided discovery and creativity. So scheme, schema and schemata become the better bets when it comes to the development of educational constructs for the Education of the Head.

Educational Implications Based Upon the Genevan School

True to the best tradition of pure mathematics and science, Piaget uttered very little on the education signifi-

cance of his vast work. Here, he need not be taken literally despite the fact that he used to say frequently, "I am not interested in psychology, education and children". It does not amount to going too far when he says that children learn through self activity. Secondly, children learn from each other in group situations. Thirdly, teachers are completely mistaken if they think that they can give an idea to their pupils; and that pupils have completely mastered the concept, if they give the right answer to the teachers' questions. They may accommodate to the new knowledge, a case of false knowlege in the phraseology of E. A. Lunzer, but it is only through their thinking and action of things, objects and ideas that they assimilate knowledge. They themselves have to be quite active in furthering their own knowlege. So he emphasizes Education for Understanding through exploration and invention of operation. Novelties of thought are created this way because teaching means creating structures which are pupil discovered rather than transmitted to pupils, available only for blurting out on askance during examinations. He appears to be silent on incentives, rewards, and motivation. In his scheme, they are not raised to the status of mental structures necessary for spontaneous development. Only certain logico mathematical structures in diverse subject matter areas in science and mathematics, social awareness, of course, not excluded, are strictly speaking, given weightage. In the Essential Piaget, Howard E. Gruber and J. Jacques Voneche schematize the following four possible educational approaches based upon the application of the Piagetian Ideas :

1. Taos : respect for the individual artisan. Abundance of semi-structured instructional and illustrative materials drawn from diverse fields of knowledge.
2. Paris : pleasures of discussion between equal partners.

3. Athens : as reflected in the Socratic method.
4. Eldorado : as reflected in inquiry and discovery methods (5).

Let us now refer to the work of other Piagetian workers. First is the Karplus Cycle of Learning. Robert Karplus and his co-workers have developed three distinct types of lessons in science which fascinate as well as fire the imagination of young children. In the first lesson, the child is asked to explore the given semi-structured materials in any way he likes. In the second lesson, he is compelled to invent the concept. In the third lesson, after the concept is fairly firmly introduced, he is expected to apply it, and thus, see its utility and futility in other situations. Thus the teaching learning situation is suitably adapted in an open manner to meet the mentality of children in the development of Piaget inspired Science Curriculum Improvement Study Project in U. S. A. Apart from the significant works of Professors Peel, Lovell, Lunzer and Beard in U.K., Dr. Wynne Harlen there has also done creditable work in matching natural environment to children's educational development the Piagetian way not only in the area of concept development in science but also in a very wide frame of reference as exemplified in the Progress in Learning Science Project for youngsters. Equally of great interest is the work of Dr. Michael Shayer which examines the conceptual demands made by a couple of scientific subjects on British G.C.E Children. Again in the Piagetian context, his work contributes to the science of science teaching in any situation seen to be a learning deficit one.

The centre now shifts to Australia. Fourthly, in the same Piagetian context, Professor K. F. Collis and Professor J. B. Biggs in their recent publication on Classroom Examples of Cognitive Thought : The Solo Taxonomy, have illustrated a rating scale where any given pupil res-

ponse evoked by a specific task each drawn from several school subjects could be placed on any one of the following points or levels, namely, Pre-structural, Uni-structural, Multi-structural Relational & Extended abstract regardless of the different levels of intellectual development: Pre-operational, Early Concrete, Middle Concrete, Concrete Generalization & Formal as defined by the Geneva school. On the very face of it, this Australian Taxonomy very clearly impinges simultaneously in the triple areas of Curriculum, Teaching Methods and Evaluation. If carefully applied, this exercise very clearly brings out the main strengths and weaknesses of pupil responses emitted in raw in the individual contexts of the various problems used in day to day classroom teaching. And when these responses are seen and examined cross-sectionally, they in a way inform the practising teachers about the properties of pupil thought processes at succeeding age levels. Whereas the Piagetian way focusses the consequent educational issues convergently, the Australian SOLO diverges, possibly the same, for the benefit of the Australian teachers to meet individual differences when pupils at different levels of development are found in the same class. And this is going to be a global classroom phenomenon.

Now coming home, not long ago, Professor J.K. Shukla talked of Need for Self Directed Change in Colleges of Education in India. Inspired by this ideal, Professor Vaidya and Dr. B. K. Sharma developed a Science Teacher Behaviour Inventory after having injected in it the Piagetian flavour. It had the following six dimensions, namely, Structuring the teaching, Structuring the learning, Structuring the materials, Structuring the classroom control, Silent Activity and Confusion. Professor Vaidya and Shri T. S. Sandhu have documented studies hinting at the distinct possibility of Hump Effect during the transitory stage between the concrete and formal stages of mental

development in the problem solving context. Prof. Vaidya and Shri S. C. Jain are presently engaged in studying advanced problem solving processes in physics. Equally worth referring to is an ERIC (NCERT) project in progress on the Determination and Development of Schemes of Thought in Science During Adolescence at the Regional College of Education, Ajmer. A chip of this project relates to the Acceleration of Thought on the Construct of Exclusion of Variables During Adolescence which is being studied by Professor Vaidya and Miss Padmini. Both of them have documented and made available a vague factorial structure of adolescent thought in the Piagetian context. Combining creativity, least understood in the Piagetian context, with the above mentioned hypothesized construct is being investigated by Mrs. Avinash Grewal at Regional College of Education, Bhopal. The Educational implications arising out of these works may not be round the corner but they will definitely go a long way in clarifying several intellectual issues in the area of methodology of teaching, particularly speaking, concept formation and problem solving as well as its modern versions like inquiry or discovery approach.

Lastly, Piaget's ancestory can be traced back to the intelligence testing at the Binet laboratory in Paris. Unlike others, he negotiated imaginatively round the Corner of Errors in assessing the nature of intelligence the other way round. With this step, he has influenced the initial, middle and end games of several workers all over the world. Like him, they are also now engaged in delineating this Piagetian territory very lovingly. Obeying the famous Inverse Square Law in Physics, nobody, beyond 10 kms. or so away from Geneva knows the full import of Piaget's work where both its substance and shadow intermingle imperceptibly. The negotiating here is becoming nearly impossible especially when Piaget is too fast in revising his ideas in the light of further reflection and criticism. All of us, therefore,

interpret His Trace in the heat of the moment. The Vision of his work, may it be a case of one sided interpretation, puts aside on the road several of the thinkers usually encountered in educational literature when it comes to the Education of the Head. In this context, what ought to be the main aim of education. Again to quote Piaget :

> The principal goal of education is to create men who are capable of doing new things, not simply repeating what other generations have done.....men who are creators, inventors and discoverers. The second goal of education is to form minds which can be critical, can verify, and do not accept everything they are offered. The great danger today is from slogans, collective opinions, readymade trends of thought. We have to be able be resist individually, to criticize, to distinguish between what is proven and what is not. So we need pupils who are active, who learn early to find out by themselves, partly by their own spontaneous activity and partly through material we set up for them, who learn early to tell what is verifiable and what is simply the first idea to come to them (12).

Scientists discovered penicillin in the mouldy bread. About 2500 years ago, Plato talked of cutting off the problems at their joints. In the process of interiorization of knowledge founded on action, Jean Piaget appears to have cut off several joints in the area of cognition by giving each of them an epistemological context within the black box as visualized by the behaviourists between Stimulus and the Response for the benefit of carrying out the needed structural improvements in the teaching-learning process by the practising teachers who may now know how to act decisively when they know how the human mind reacts to concrete and abstract situations at the various levels of

intellectual development implying thereby that more and more inputs in education may not lead to more and more astounding results for vice versa may even hold true in the Genevan context. So, in the name of modernization, if any country simply down grades the content, it is bound to generate allergies all around within its borders for the basic Piagetian essence process at the cost of Product is just missing in this scheme. In summary, spin off advantages, await round the corner if one cares to turn one's gaze on the Genevan School so laboriously built imaginatively under the leadership of Professor Jean Piaget, the polymath when he said :

The fundamental hypothesis of genetic epistemology is that there is a parallelism between the progress made in the logical and rational organization of knowledge and the corresponding formative logical processes. Of course, the most fruitful field of study would be reconstituting human history—the history of human thinking in prehistoric man. Since this field of biogenesis is not available to us, we shall do as biologists do and turn ontogenesis.

From this, according to A. Blanchet and others, emerges the finest educational principle meeting diverse individual differences of the growing youngsters : It Must Permit the Child to Establish Plans to Reach Distant Goals, While Leaving him Wide Open Freedom to Follow His Own Routine. The gap between the learning in the classroom and outside of it is closed for parts of the hidden environment open up as naturally as Learning to Walk through the three lines of Access to Knowledge, namely, Perceptual, Acting and Conceptual. The world need no longer pass by as usual for it is the business of the human mind to go on creating novelties, each more noveler than the other, for we do participate in the intellectual race trailed by Einstein, Freud, Chomsky and Piaget, to mention a few in this century.

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The Puzzling Ways of Science*

For all practical purposes, Michael Faraday was an illiterate scientist. Through hit and trial, he discovered the laws of electromagnetic induction as well as electrolysis. Interestingly enough, he also placed his head between the poles of a strong magnet for determining the effect of magnetism on physiology. Several years later, this inspired Professor Zeeman to place a spectral line under strong magnetic fields and the result was a great discovery. He got a Nobel prize for Zeeman effect. Professor Patten koffer of Germany, swallowed two phials of deadly cholera germs and did not contract cholera. Why ? Because he had not willed so. The concept of immunity was born later on. In the beginning of this century, Prof. Thomson got a Nobel prize for showing that electrons were particles. About thirty-five years later, his son, using methods of science, showed that electrons were waves. Both were awarded Nobel prizes for presenting two opposing Views. Professor Kepule of Germany discovered the structure of Benzene not in the laboratory but in the dream. It need not be mentioned that Dalton cooked up data for suggesting firmly the theory of atomic structure.

The puzzling ways of science do not end here. A very well known chemist of the 17th century named Jan Van Helmount gave us the following receipt for preparing mice: 'Take slight soiled shirts, place them in a dim, quiet corner, sprinkle wheat and in 21 days, one would have mice. The experiment was repeated by scientists in the whole of Europe for about 75 years. Surprisingly enough, each got mice with this formula. Failure in science is also not a rare event. The Michelson-Morley experiment showed

that the earth revolves round the sun. But the concept of Ether was rejected. Sometimes, scientists view themselves very differently. Harold Urey, another Nobel prize winner, would have spent his life as a farmer had he got one mark less than the pass mark. Each one of us knows that a magnet points towards north-south. Does it always point in the north south direction ? The answer to this question was given unsuccessfully by Michael Faraday, Lord Kelvin. Lord Belfour and others. Lord Appleton got a Nobel prize for providing the right explanation.

Lastly, scientists when faced with problems try hard to solve them by forming and testing hypotheses. They consider many possibilities for solving a problem. It does not matter if they end up with wrong results for they are corrected. In the words of Prof. J. W. Debye.

> It is the urge to understand that makes me study things. Perhaps in an experiment, something happens that astonishes me. I think about what it could mean. Try this, try that in my mind. Discontent pushes me. It is the discontent that is the driving force. Soon I become impatient and then I get nowhere so I forget about it and then months later, answre suddenly comes to me. Once I have it and tested that it is the right answer, I lose interest. I then want to explore something new. So uncovering science is more different than covering science and hence the puzzling ways of science.

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*Contributed by Shailendra Vaidya. Reproduced from the DMPHS Magazine PRERANA, 12th issue, 1979-80.

Characteristics of Abstract as Compared to Concrete Thought*

Concrete Operations

1. Classifications and serial ordering are employed in a step-by-step fashion, without relating all of the links into a system.
2. Well-mixed variables in an investigation cannot be separated.
3. Logical multiplication of factors is limited to 1 by 1 or 2 by 2 correspondences but a total n by n system is not yet available.
4. Reversibility in operations is limited to negation or reciprocity but they are not interrelated as a system.
5. Experimental variability is seen as a result of multiple causes and no systematic efforts are made to isolate and control factors or variables.
6. Chance or probability ideas are influenced by previous results, yielding a kind of "gamblers' fallacy."
7. The notion of correlation is incomplete, and errors occur when anything other than single relationships is involved.
8. Proportions are not as yet available except in the simplest sense.

Formal or Abstract Operations

1. Reasoning now begins with propositions and hypotheses. A system of what is hypothetically possible is structured and followed by empirical verification.
2. Factors or variables in an experimental setting are now seen as distinct and can be separated from one another.
3. A complete combinatorial system is available so that all combinations of factors (n by n) can be exhaustively tested.
4. Reversibility is advanced by the inter-relationship of negation and reciprocity, resulting in the ability to maintain a dynamic equilibrium in a system involving many factors.
5. Variables can now be controlled systematically.
6. The notion of probability becomes operational and widely applicable.
7. Correlations are no longer limited to 1 or 1/2, but are, at least qualitatively also applicable in such situations as 1/6, 2/7, 3/11, and so on.
8. Proportions are now much more widely applicable in solving problems.

*Reproduced from How Children Learn Science by Ronald G. Good

9. Experimental contradictions cannot be pursued in a systematic manner.
10. Conservation is limited to those physical qualities that are easily tested.
11. Thinking is a derivative of the child's own actions on concrete reality.
12. "Models" represent the concrete reality that seems to be offered rather than abstract possibilities worked out by others.

9. Because of the complete combinatorial system experimental contradictions can now be isolated and solved.
10. Conservation is no longer limited to the immediate testable environment.
11. Coordination of reference systems is possible and acceleration is worked out.
12. In an experimental situation involving many variables, logical pairs are sorted out and tested.
13. The utilization of a proof based on "all other thing being equal" is now pursued.
14. The binary system of propositional logic based on the operations of conjunction, disjunction, implication and others are operable.
15. The INRC group transforms the binary system into a fully operational interrelated system.

Age Ranges Suggested for Certain Piagetian Concepts*

Classification	Attained at Age Range
a. Intensive—extensive coordination	7-9
b. Stable classification criteria	7-9
c. All—some relations	8-10
d. Class inclusion	9-11
Seriation	
a. Simple series	5-7
b. Insert elements into series	7-9
c. Analyze elements using whole series shape	9-10
Number	
a. Conserver	6-8
b. Coordination of ordinal and cardinal properties	7-9
c. Singular class	8-9
d. Null class	9-11
Topology	
a. Rhombus reconstruction	6-7
b. Linear and circular order	7-8
c. Knot tying	8-10
d. Continuity	10-12
Projective Geometry	
a. Straight line construction	7-8
b. Perspective	
Simple objects	8-9
Complex structures	10-11
c. 3-Dimentional Visualization	
Simple structures	7-8
Complex structures	
d. Shapes from cutting solids	10-12
Simple	7-8
Complex	11-12
Euclidean Geometry	
a. Water-level conservation	9-11

* Reproduced from How Children Learn Science by Ronald G. Good.

b. Conservation of parallelism	9-11
c. Conservation of angles	10-12
d. Metric proportions	12-16
Length and Distance	
a. Conservation of distance (empty space)	7-8
b. Conservation of length (filled space)	6-8
c. Iteration of length	8-9
d. Subdivision of Straight lines	
Staggered but equal	7-9
Staggered but unequal	8-10
Area	
a. Conservation of area	8-11
b. Measurement by iteration	8-11
c. Measurement by superposition	9-11
d. Perimeter-area relationship	12-
Volume	
a. Conservation of internal volume	9-11
b. Conservation of dispalacement volume	11-15
Coordinate systems	
a. Generalized reference system	11-14
b. Angle measurement	10-12
Time and Motion	
a. Co-seriation of sequence and duration	8-10
b. Succession and simultaneity	8-10
c. Transivity of equal and unequal durations	8-10
d. Time measurement	
Linear motion	9-10
Circular motion	11-14
e. Relative speed	11-14
f. Conservation of uniform speed	10-12
g. Acceleration	12-
Mass, Weight and Density	
a. Conservation of continuous quantity	8-10
b. Conservation of weight	10-12
c. Density	12-
d. Weight—distance equilibration	12-

The Modification of Concrete Structures Into Formal Structures*

Five tranformations mark the passage from the concrete operational level of thought to the stage of formal operations.

1. The first and most important transformation is the capacity for reasoning on hypotheses. This mode of reasoning, often called hypothetico-deductive, is used by scientists in their work: formulate certain hypotheses about the nature of the universe, deduce some logical consequences from these hypotheses, then observe the universe to whether it behaves according to expectations.

To a superficial observer, the concrete operational subject seems capable of reasoning on hypotheses, since he uses various "maybes" to account for his observations. But, in the end, these numerous "maybes" boil down to mere fictions that are submitted neither to verification nor experimental proof.

Without hypothetical thinking the idea of proof has no meaning. It happens that some fortunate preadolescents grasp the idea of proof at the very moment in their development that they are exposed to the system of proofs of Euclidean geometry. This can be an exciting moment in life.

2. The second transformation is the use of propositional logic. At the concrete operational level, the child is capable of reasoning on what Piaget calls interpropositional operations. He has become capable of evaluating the validity of a train of reasoning independently of its factual content.

He has become interested in establishing the logical link between premises and conclusions independently of the nature of these propositions.

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* Quoted from The Essential Piaget

3. This new capacity leads to the third transformation marking this new stage of development, the capacity to dissociate completely from contents, which is the logical consequence of the second transformation. The preadolescent is capable of replacing any concrete proposition by an arbitrary sign such as the 'p' and 'q' of symbolic logic, he is also capable of replacing the concrete relations between propositions by all sorts of cablistic signs that logicians invent every day for their apparent utmost satisfaction.

4. The fourth transformation deals with the combinatorial nature of operations. Given two propositions, 'p' and 'q', that can be either true or false, the concrete operational child is capable of grouping them into four types of associations : (1) both true, (2) both false; (3) 'p' true and 'q' false; (4) 'p' false and 'q' true. For Piaget this elementary form of association differs fundamentally from true combination which would fully develop the sixteen subsets of the four associations of two initial propositions. This combinatorial system can be extended from the sixteen binary operations to the 256 ternary operations. The point of this combinatorial system is that the operations in question form a closely knit system in which passage from one element of the structure to another is always possible.

5. Thanks to this closed system of transformations, the preadolescent becomes capable of inserting real cases in the set of all possible cases that can be generated logically. Reality has become a special case of possibility. The system of all possible combinations forms the logic of propositions whose use and mastery constitute the fifth basic transformation of the formal operational stage of thought.

For Piaget, these five transformations derive from one single identifiable mental structure : the INRC group where I stands for identity, N for negation, R for recipro-

city, and C for correlative or dual operation. Many things can be described with these operations, for instance, the functioning of an hydraulic press where I would stand for the pressure of the piston; N for the elimination of the pressure; R for the resistance of the pressurized liquid and C for the elimination of the resistance. These four operations form a group; there is a complete passage from one element of the system to another and the result of such an operation is always an element of the group. 'According to Piaget, this fascinating new mental structure results from the fusion of two different structures, the lattice and the group of reversibilities, into one coherent structure, the INRC group.'

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